4/-



To Miss Marion Gray
With kind love
from
Robt Watt

Burnside House
23 Jany 1895

# A TREATISE OF PRACTICAL GEOMETRY.

IN THREE PARTS.

By the late Dr DAVID GREGORY,

Sometime Professor of Mathematics in the University of EDINBURGH, and afterwards *Savilian* Professor of Astronomy at OXFORD.

[Translated from the Latin. With Additions.]

The SEVENTH EDITION.

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M,DCC,LXIX.

# PREFACE,

THIS Treatiſe was compoſed in Latin, about ſixty Years ago, by Dr DAVID GREGORY, then Profeſſor of Mathematics in the Univerſity of Edinburgh; where it has been conſtantly taught ſince that time, immediately after Euclid's Elements and the plain Trigonometry, as proper for exerciſing the Students in the Application of Geometry to Practice. The Bookſeller having procured an Engliſh Tranſlation of it, which had been made by an ingenious Gentleman when a Student here, this Tranſlation has been reviſed;

revised; and several Additions have been made to the treatise itself, in order to render it more useful at this time. The Reader will find these distinguished from the Author's Text.

Col. McLaurin.

College of Edinb.
May 1. 1745.

# A TREATISE OF PRACTICAL GEOMETRY.

HAVING explained the firſt ſix books of Euclid, with the eleventh and twelfth, which may ſerve for geometrical elements; and having alſo taught the plain Trigonometry; we are now to ſubjoin ſome corollaries which are eaſily deduced from them, that contain practical rules of great uſe in the affairs of life, concerning the menſuration of lines, angels, ſurfaces, and ſolids.

This Treatiſe of Practical Geometry is divided into three parts. In the firſt,

we treat of the menſuration of lines and angles; to which we have ſubjoined Surveying. In the ſecond, we treat of ſurfaces; not of ſuch as are plain only, but of ſome curve ſurfaces likewiſe; as of the ſurface of the cylinder, cone, and ſphere; and of thoſe parts of the ſphere which we have frequently occaſion to conſider. It is ſhewn how to expreſs the area of theſe in the ſuperficial meaſures that are now in uſe amongſt us. The third part treats of ſolid figures and their menſuration. After deducing the rules for finding the ſolid content of the parallelopipedon, priſm, pyramid, cylinder, cone, &c. from Euclid, we add, from Archimedes, the menſuration of the ſphere and ſpheroid, and of their ſegments, demonſtrated in an eaſy manner; from whence a method is derived for finding the contents of veſſels that are either full, or in part empty, in the wet as

well

well as the dry meaſures, that are now in uſe amongſt us.

## PART I.

A Line, or length, to be meaſured, whether it be diſtance, height, or depth, is meaſured by a line leſs than it. With us the leaſt meaſure of length is an inch: not that we meaſure no line leſs than it, but becauſe we do not uſe the name of any meaſure below that of an inch; expreſſing leſſer meaſures by the fractions of an inch: and in this treatiſe we uſe decimal fractions as the eaſieſt. Twelve inches make a foot; three feet and an inch make the Scots ell; ſix ells make a fall; forty falls make a furlong; eight furlongs make a mile: ſo that the Scots mile is 1184 paces, accounting every pace to be five feet. Theſe things are according to the ſtatutes of Scotland; notwithſtanding which, the glaziers uſe a foot of

of only eight inches; and other artiſts for the moſt part uſe an Engliſh foot, on account of the ſeveral ſcales marked on the Engliſh foot-meaſure for their uſe. But the Engliſh foot is ſomewhat leſs than the Scots; ſo that 185 of theſe make 186 of thoſe.

Lines, to the extremities and any intermediate point of which you have eaſy acceſs, are meaſured by applying to them the common meaſure a number of times. But lines, to which you cannot have ſuch acceſs, are meaſured by methods taken from Geometry; the chief whereof we ſhall here endeavour to explain. The firſt is by the help of the Geometrical ſquare.

" As for the Engliſh meaſures, the
" yard is three feet, or thirty-ſix inches.
" A pole is ſixteen feet and a half, or
" five yards and a half. The chain,
" commonly called Gunter's chain, is
" four poles, or twenty two yards, that
" is, ſixty-ſix feet. An Engliſh ſtatute mile

" is

" is fourſcore chains, or 1760 yards, " that is, 5280 feet.

The chain (which is now much in " uſe, becauſe it is very convenient for " ſurveying) is divided into a hundred " links, each of which is $7\frac{92}{100}$ of an " inch: whence it is eaſy to reduce any " number of thoſe links to feet, or any " number of feet to links.

" A chain that may have the ſame " advantages in ſurveying in Scotland, " as Gunter's chain has in England, ought " to be in length ſeventy four feet, or " twenty four Scots ells, if no regard is " had to the difference of the Scots and " Engliſh foot above mentioned. But, " if regard is had to that difference, " the Scots chain ought to conſiſt of " $74\frac{2}{5}$ Engliſh feet, or 74 feet 4 inches " and $\frac{4}{5}$ of an inch. This chain being " divided into an hundred links, each " of thoſe links is 8 inches and $\frac{928}{1000}$ " of an inch. In the following table, " the moſt noted meaſures are expreſ-

" ſed

" sed in English inches and decimals of " an inch."

| | English Inch. | Dec. |
|---|---|---|
| The English foot, is - - | 12 | 000 |
| The Paris foot, - - | 12 | 788 |
| The Rhinland foot, measured by Mr Picart, - - | 12 | 362 |
| The Scots foot, - - | 12 | 065 |
| The Amsterdam foot, by Snellius and Picart, - | 11 | 172 |
| The Dantzick foot, by Hevelius, | 11 | 297 |
| The Danish foot, by Mr Picart, | 12 | 465 |
| The Swedish foot, by the same, | 11 | 692 |
| The Brussels foot, by the same, | 10 | 828 |
| The Lyons foot, by Mr Auzout, | 13 | 458 |
| The Bononian foot, by Mr Cassini, | 14 | 938 |
| The Milan foot, by Mr Auzout, | 15 | 631 |
| The Roman palm used by merchants, according to the same, - - | 9 | 791 |
| The Roman palm used by architects, - - | 8 | 779 |
| The palm of Naples, according to Mr Auzout, - | 10 | 314 |
| The Enlish yard, - | 36 | 000 |

The

| | Inch. | Dec. |
|---|---|---|
| The English ell, - | 45 | 000 |
| The Scots ell, - - | 37 | 200 |
| The Paris aune used by mercers, according to Mr Picart, | 46 | 786 |
| The Paris aune used by drapers, according to the same, | 46 | 680 |
| The Lyons aune, by Mr Auzout, | 46 | 570 |
| The Geneva aune, - | 44 | 760 |
| The Amsterdam ell, - | 26 | 800 |
| The Danish ell, by Mr Picart, | 24 | 930 |
| The Swedish ell, - | 23 | 380 |
| The Norway ell, - | 24 | 510 |
| The Brabant or Antwerp ell, | 27 | 170 |
| The Brussels ell, - | 27 | 260 |
| The Bruges ell, - | 27 | 550 |
| The brace of Bononia, according to Auzout, - - | 25 | 200 |
| The brace used by architects in Rome, - - | 30 | 730 |
| The brace used in Rome by merchants, - - | 34 | 270 |
| The Florence brace used by merchants, according to Picart, | 22 | 910 |
| The Florence geographical brace, | 21 | 570 |

The

| | Inch. | Dec. |
|---|---|---|
| The vara of Seville, - | 33 | 127 |
| The vara of Madrid, - | 39 | 166 |
| The vara of Portugal, - | 44 | 031 |
| The cavedo of Portugal, - | 27 | 354 |
| The ancient Roman foot, - | 11 | 632 |
| The Persian arish, according to Mr Greaves, - | 38 | 364 |
| The shorter pike of Constantinople, according to the same, | 25 | 576 |
| Another pike of Constantinople, according to Mess. Mallet and De la Porte, - - | 27 | 920 |

## PROPOSITION I.

### PROBLEM I.

*To describe the structure of the Geometrical square.*

THE Geometrical square is made of any solid matter, as brass or wood, or of any four plain rulers joined together

gether at right angles, (as in Fig. 1.); where A is the centre, from which hangs a thread with a ſmall weight at the end, ſo as to be directed always to the centre. Fach of the ſides BE and DE is divided into an hundred equal parts, or (if the ſides be long enough to admit of it) into a thouſand parts; C and F are two ſights, fixed on the ſide AD. There is moreover an index GH, which, when there is occaſion, is joined to the centre A, in ſuch manner as that it can move round, and remain in any given ſituation. On this index are two ſights perpendicular to the right line going from the centre of the inſtrument: theſe are K and L. The ſide DE of the inſtrument is called the upright ſide; BE the reclining ſide.

## PROP. II. FIG. 2.

*To measure an accessible height,* AB, *by the help of a Geometrical square, its distance being known.*

LET BR be an horizontal plane, on which there stands perpendicularly any line AB: Let BD, the given distance of the observator from the height, be 96 feet; let the height of the observator's eye be supposed 6 feet; and let the instrument, held by a steady hand, or rather leaning on a support, be directed towards the summit A, so that one eye (the other being shut) may see it clearly through the sights; the perpendicular or plum-line mean while hanging free, and touching the surface of the instrument: Let now the perpendicular be supposed to cut off on the right side KN 80 equal parts. It is clear that LKN, ACK, are similar triangles; for the angles LKN, ACK, are

are right angles, and therefore equal: moreover LN and AC are parallel, as being both perpendicular to the horizon; consequently, by Prop. 29. 1. B. of Euclid, the angles KLN, KAC, are equal; wherefore, by the second corollary and of the 32. Prop. 1. B. of Euclid, the angles LNK, and AKC, are likewise equal: So that in the triangles NKL, KAC, (by the 4. Prop. of the 6. B. of Euclid) as NK : KL :: KC (*i. e.* BD) : CA; that is, as 80 to 100, so is 96 feet to CA. Therefore, by the rule of three, CA will be found to be 120 feet; and CB, which is 6 feet, being added, the whole height is 126 feet.

But if the distance of the observator from the height, as BE, be such, that, when the instrument is directed as formerly toward the summit A, the perpendicular falls on the angle P, opposite to H, the centre of the instrument, and BE or CG be given of 120 feet;

CA

CA will alſo be 120 feet. For in the triangles HGP, ACG, aequiangular, as in the preceeding caſe, as PG : GH : : GC : CA. But PG is equal to GH; therefore GC is likewiſe equal to CA: that is, CA will be 120 feet, and the whole height 126 feet as before.

Let the diſtance BF be 300 feet, and the perpendicular or plum-line cut off 40 equal parts from the reclining ſide: Now, in this caſe, the angles QAC, QZI, are equal, by the 29. Prop 1. B. of Euclid. And, by the ſame Prop. the angles QZI, ZIS, are equal; therefore the angle ZIS, is equal to the angle QAC. But the angles ZSI, QCA are equal, being right angles; therefore in the aequiangular triangles ACQ, SZI, by the 4. Prop. of the 6. B. of Euclid, it will be, as ZS : SI : : CQ : CA; that is, as 100 to 40, ſo is 300 to CA. Wherefore, by the rule of three, CA will be found to be of 120 feet. And, by adding the height of the obſervator, the whole

whole BA will be 126 feet. Note, That the height is greater than the diſtance, when the perpendicular cuts the right ſide, and leſs, if it cut the reclined ſide; and that the height and diſtance are equal, if the perpendicular fall on the oppoſite angle.

### *SCHOLIUM.* Fig. 3.

If the height of a tower to be meaſured as above, end in a point, as in Fig. 3. the diſtance of the obſervator oppoſite to it, is not CD, but is to be accounted from the perpendicular to the point A; that is, to CD muſt be added the half of the thickneſs of the tower, viz. BD: Which muſt likewiſe be underſtood in the following propoſitions, when the caſe is ſimilar.

PROP.

## PROP. III.

### PROB. FIG. 4.

*From the height of a tower* AB *given, to find a distance on the horizontal plane* BC, *by the Geometrical square.*

LET the instrument be so placed, as that the mark C in the opposite plane may be seen through the sights; and let it be observed how many parts are cut off by the perpendicular. Now, by what hath been already demonstrated, the triangles AEF, ABC, are similar; therefore, by 4th, 6. Eucl. it will be as EF to AE, so AB (composed of the height of the tower BG, and of the height of the centre of the instrument A, above the tower BG) to the distance BC. Wherefore, if, by the rule of three, you say, as EF to AE, so is AB to BC, it will be the distance sought.

## PROP. IV. Fig. 5.

*To measure any distance at land or sea, by the Geometrical square.*

IN this operation, the index is to be applied to the instrument, as was shown in the description; and, by the help of a support, the instrument is to be placed horizontally at the point A; then let it be turned till the remote point F, whose distance is to be measured, be seen through the fixed sights; and bringing the index to be parallel with the other side of the instrument, observe by the sights upon it any accessible mark B, at a sensible distance: then carrying the instrument to the point B, let the immoveable sights be directed to the first station A, and the sights of the index to the point F. If the index cut the right side of the square, as in K, in the two triangles BRK, and BAF, which are aequiangular,

lar, it will be (by 4th 6. Eucl.) as BR to RK, so BA (the distance of the stations to be measured with a chain) to AF; and the distance AF sought will be found by the rule of three. But if the index cut the reclined side of the square in any point L, where the distance of a more remote point is sought; in the triangles BLS, BAG, the side LS shall be to SB, as BA to AG, the distance sought; which accordingly will be found by the rule of three.

## PROP. V.

### PROB. FIG. 6.

*To measure an accessible height by means of a plain Mirror.*

LET AB be the height to be measured; let the Mirror be placed at C, in the horizontal plane BD, at a known distance BC; let the observer go back to D, till he see the image of the

the ſummit in the mirror, at a certain point of it, which he muſt diligently mark; and let DE be the height of the obſervator's eye. The triangles ABC and EDC are æquiangular; for the angles at D and B are right angles; and ACB, ECD, are equal, being the angles of incidence and reflexion of the ray AC, as is demonſtrated in optics; wherefore the remaining angles at A, and E, are alſo equal: Therefore, by 4th, 6. Eucl. it will be, as CD to DE, ſo CB to BA; that is, as the diſtance of the obſervator from the point of the mirror in the right line betwixt the obſervator and the height, is to the height of the obſervator's eye, ſo is the diſtance of the tower from that point of the mirror, to the height of the tower ſought; which therefore will be found by the rule of three.

Note .. The obſervator will be more exact, if, at the point D, a ſtaff be placed in the ground perpendicularly, over

the top of which the obſervator may ſee a point of the glaſs exactly in a line betwixt him and the tower.

Note 2. In place of a mirror may be uſed the ſurface of water contained in a veſſel, which naturally becomes parallel to the horizon.

## PROP. VI. FIG. 7.

*To meaſure an acceſſible height* AB *by means of two ſtaffs.*

LET there be placed perpendicularly in the ground a longer ſtaff DE, likewiſe a ſhorter one FG, ſo as the obſervator may ſee A, the top of the height to be meaſured, over the ends D, F, of the two ſtaffs; let FH and DC, parallel to the horizon, meet DE and AB in H and C; then the triangles FHD, DCA, ſhall be æquiangular; for the angles at C and H are right ones; likewiſe the angle

angle A is equal to the angle FDH, by 29. 1. Eucl.; wherefore the remaining angles DFH, and ADC, are alſo equal: Wherefore, by 4. 6. Eucl. as FH, the diſtance of the ſtaffs, to HD, the exceſs of the longer ſtaff above the ſhorter; ſo is DC, the diſtance of the longer ſtaff from the tower, to CA, the exceſs of the height of the tower above the longer ſtaff. And thence CA will be found by the rule of three.

To which if the length DE be added, you will have the whole height of the tower BA. Q. E. F.

## *SCHOLIUM.* FIG. 8.

Many other methods may be occaſionally contrived for meaſuring an acceſſible height. For example, from the given length of the ſhadow BD, I find out the height AB, thus: Let there be erected a ſtaff CE perpendicularly, producing the ſhadow EF: The triangles ABD, CEF, are æquiangular; for the

the angles at B, and E, are right; and the angles ADB, and CFE, are equal, each being equal to the angle of the ſun's elevation above the horizon: Therefore, by 4th, 6. Eucl. as EF, the ſhadow of the ſtaff, to EC, the ſtaff itſelf, ſo BD, the ſhadow of the tower, to BA, the height of the tower. Though the plane on which the ſhadow of the tower falls be not parallel to the horizon, if the ſtaff be erected in the ſame plane, the rule will be the ſame.

## PROP VII.

*To meaſure an inacceſſible height by means of two ſtaffs.*

HITHERTO we have ſuppoſed the height to be acceſſible, or that we can come at the lower end of it; now if, becauſe of ſome impediment, we cannot get to a tower, or if the point whoſe height is to be found out be the ſummit of a hill, ſo that the perpendicular

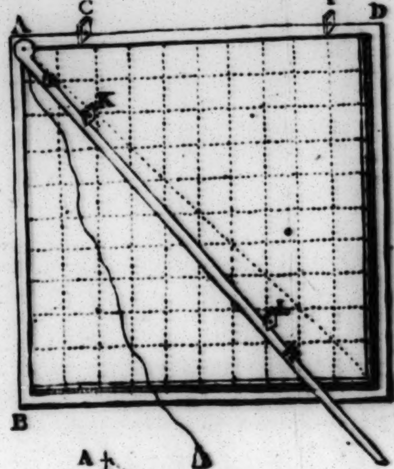
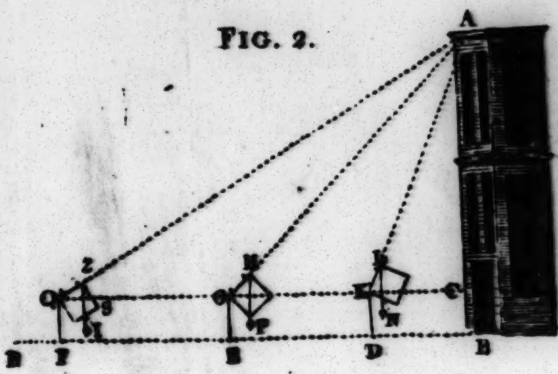


FIG. 1.

FIG. 2.



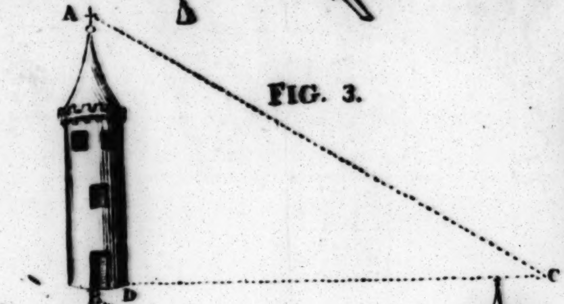
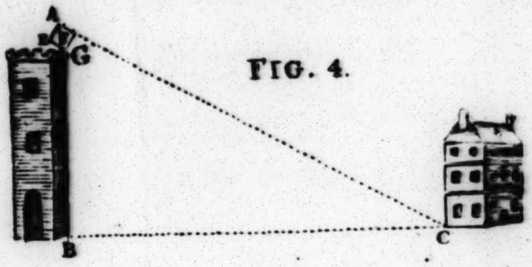


FIG. 3.

FIG. 4.



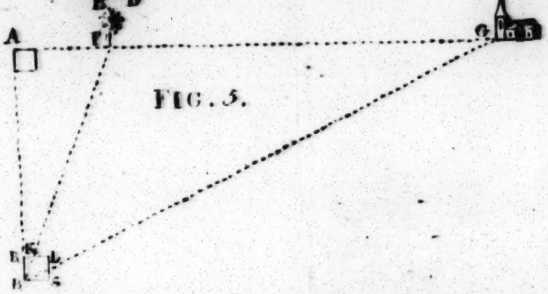


FIG. 5.

FIG. 6.

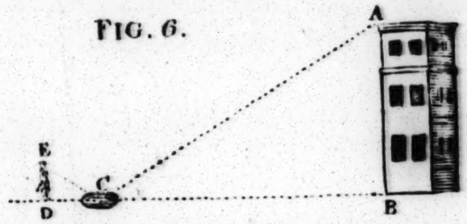


FIG. 7.

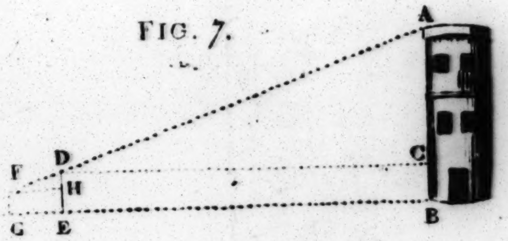


FIG. 8.

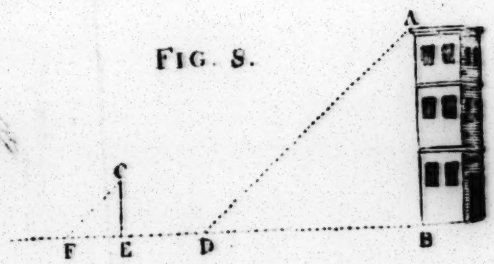


FIG 10

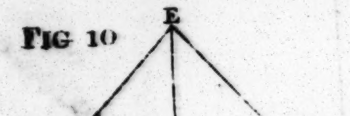
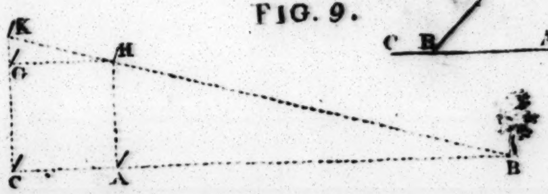


FIG. PROP. 7.

FIG. 9.



pendicular he hid within the hill; if, I ſay, for want of better inſtruments, ſuch an inacceſſible height is to be meaſured by means of two ſtaffs, let the firſt obſervation be made with the ſtaffs DE and FG, as in Prop. VI.; then the obſervator is to go off in a direct line from the height and firſt ſtation, till he come to the ſecond ſtation; where he is to place the longer ſtaff perpendicularly at RN, and the ſhorter ſtaff at KO, ſo that the ſummit A may be ſeen along their tops; that is, ſo that the points KNA may be in the ſame right line. Through the point N let there be drawn the right line NP parallel to FA: Wherefore in the triangles KNP, KAF, the angles KNP, KAF are equal, by the 29. 1. Eucl. alſo the angle AKF is common to both; conſequently the remaining angle KPN is equal to the remaining angle KFA. And therefore, by 4th, 6. Eucl. PN : FA : : KP : KF. But the triangles PNL, FAS are ſimilar;

ſimilar; therefore, by 4th, 6. Eucl. PN : FA :: NL : SA. Therefore, by the 11. 5. Eucl. KP : KF :: NL : SA. Thence, alternately, it will be, as KP (the exceſs of the greater diſtance of the ſhort ſtaff from the long one above its leſſer diſtance from it) to NL, the exceſs of the longer ſtaff above the ſhorter; ſo KF, the diſtance of the two ſtations of the ſhorter ſtaff, to SA the exceſs of the height ſought above the height of the ſhorter ſtaff. Wherefore SA will be found by the rule of three. To which let the height of the ſhorter ſtaff be added, and the ſum will give the whole inacceſſible height BA. Q. E. F.

Note 1. In the ſame manner may an inacceſſible height be found by a geometrical ſquare, or by a plain ſpeculum. But we ſhall leave the rules to be found out by the ſtudent, for his own exerciſe.

Note 2. That by the height of the ſtaff

ſtaff we underſtand its height above the ground in which it is fixed.

Note 3. Hence depends the method of uſing other inſtruments invented by geometricians; for example, of the geometrical croſs: And if all things be juſtly weighed, a like rule will ſerve for it as here. But we incline to touch only upon what is moſt material.

## PROP. VIII. FIG. 9.

*To meaſure the diſtance* AB, *to one of whoſe extremities we have acceſs, by the help of four ſtaffs.*

LET there be a ſtaff fixed at the point A; then going back at ſome ſenſible diſtance in the ſame right line, let another be fixed in C, ſo as that both the points A and B be covered and hid by the ſtaff C; likewiſe going off in a perpendicular from the right line CB, at the point A, (the method of doing which ſhall be ſhown in the following *ſcholium*), let there be placed another ſtaff at H; and

and in the right line CKG (perpendicular to the ſame CB, at the point B), and at the point of it K, ſuch that the points K, H, and B, may be in the ſame right line, let there be fixed a fourth ſtaff. Let there be drawn, or let there be ſuppoſed to be drawn, a right line HG parallel to CA. The triangles KGH, HAB, will be æquiangular; for the angles HAB, KGH are right angles. Alſo by 29th, 1. Eucl. the angles ABH, KHG are equal; wherefore, by the 4th, 6. Eucl. as KG (the exceſs of CK above AH) to GH, or to CA, the diſtance betwixt the firſt and ſecond ſtaff; ſo is AH, the diſtance betwixt the firſt and third ſtaff, to AB the diſtance ſought.

## *SCHOLIUM.* FIG. 10.

To draw on a plane a right line AE perpendicular to CH, from a given point A; take the right lines AB, AD, on each ſide equal; and in the points

B

B and D, let there be fixed ſtakes, to which let there be tied two equal ropes BE, DE, or one having a mark in the middle, and holding in your hand their extremities joined, (or the mark in the middle, if it be but one), draw out the ropes on the ground; and then, where the two ropes meet, or at the mark, when by it the rope is fully ſtretched, let there be placed a third ſtake at E; the right line AE will be perpendicular to CH in the point A, by 11th, 1. Eucl. In a manner not unlike to this, may any problems that are reſolved by the ſquare and compaſſes, be done by ropes and a cord turned round as a radius.

## PROP. IX. FIG. 11.

*To meaſure the diſtance* AB, *one of whoſe extremities is acceſſible.*

FROM the point A, let the right line AC of a known length be made perpendicular to AB, (by the preceeding

*ſcholium)* : likewiſe draw the right line CD perpendicular to CB, meeting the right line AB in D: then, by the 8. 6. Eucl. as DA : AC :: AC : AB. Wherefore, when DA and AC are given, AB will be found by the rule of three. Q. E. F.

## *SCHOLIUM.*

All the preceeding operations depend on the equality of ſome angles of triangles, and on the ſimilarity of the triangles ariſing from that equality. And on the ſame principles depend innumerable other operations which a geometrician will find out of himſelf, as is very obvious. However, ſome of theſe operations require ſuch exactneſs in the work, and without it are ſo liable to errors, that, *cæteris paribus*, the following operations, which are performed by a trigonometrical calculation, are to be preferred; yet could we not omit thoſe above, being moſt eaſy

in

in practice, and most clear and evident to those who have only the first elements of Geometry. But if you are provided with instruments, the following operations are more to be relied upon. We do not insist on the easiest cases to those who are skilled in plain trigonometry, which is indeed necessary to any one who would apply himself to practice. It would be easy to the reader to find examples; and we have shown in plain Trigonometry how to find the angle or side of any plain triangle that is required, from the angles or sides that may be given.

## PROP. X. FIG. 12.

*To describe the construction and use of the Geometrical Quadrant.*

THE Geometrical Quadrant is the fourth part of a circle divided into ninety degrees, to which two sights are adapted, with a perpendicular

lar or plumb line hanging from the centre. The general uſe of it is for inveſtigating angles in a vertical plane, comprehended under right lines going from the centre of the inſtrument, one of which is horizontal, and the other is directed to ſome viſible point. This inſtrument is made of any ſolid matter as wood, copper, &c.

### PROP. XI. FIG. 13.

*To deſcribe the make and uſe of the graphometer.*

THE Graphometer is a ſemicircle made of any hard matter, of wood, for example, or braſs, divided into 180 degrees; ſo fixed on a *fulcrum*, by means of a braſs ball and ſocket, that it eaſily turns about, and retains any ſituation; two ſights are fixed on its diameter. At the centre there is commonly a magnetical needle in a box. There is likewiſe a moveable ruler,

ler, which turns round the centre, and retains any ſituation given it. The uſe of it is to obſerve any angle, whoſe vertex is at the centre of the inſtrument in any plane, (though it is moſt commonly horizontal, or nearly ſo), and to find how many degrees it contains.

## PROP. XII.

### FIG. 14. and 15.

*To deſcribe the manner in which angles are meaſured by a Quadrant or Graphometer.*

LET there be an angle in a vertical plane, comprehended between a line parallel to the horizon HK, and the right line RA, coming from any remarkable point of a tower or hill, or from the ſun, moon, or a ſtar. Suppoſe that this angle RAH is to be meaſured by the quadrant: let the inſtrument be placed in the vertical plane, ſo as that the centre A may be in the angular

angular point: and let the ſights be directed towards the object at R, (by the help of the ray coming from it, if it be the ſun or moon, or by the help of the viſual ray, if it is any thing elſe), the degrees and minutes in the arch BC cut off by the perpendicular, will meaſure the angle RAH required. For, from the make of the quadrant, BAD is a right angle; therefore BAR is likewiſe right, being equal to it. But, becauſe HK is horizontal, and AC perpendicular, HAC will be a right angle; and therefore equal alſo to BAR. From thoſe angles ſubtract the part HAB that is common to both; and there will remain the angle BAC equal to the angle RAH. But the arch BC is the meaſure of the angle BAC; conſequently, it is likewiſe the meaſure of the angle RAH.

Note, That the remaining arch on the quadrant DC is the meaſure of the angle RAZ, comprehended between the

FIG. 11.

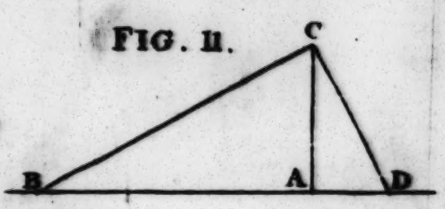



FIG. 12.

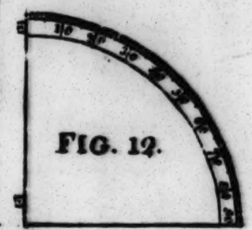


FIG. 13.

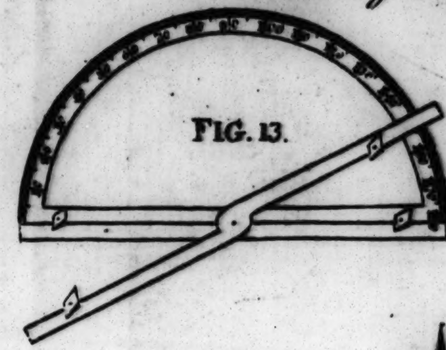


FIG. 14.

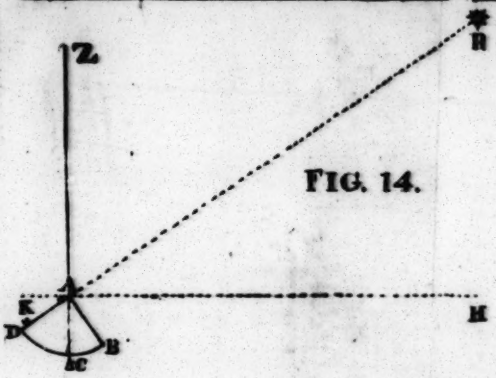



FIG. 15.

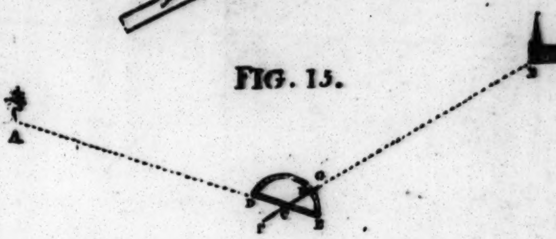



FIG. 16.

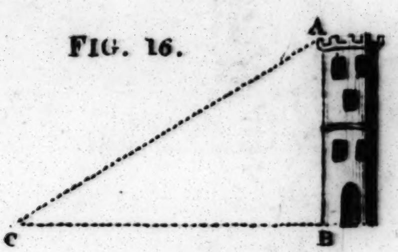



FIG. 17.

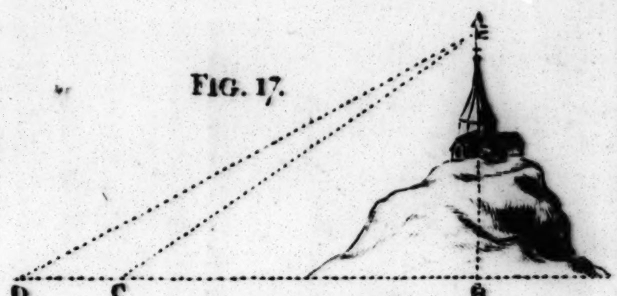



FIG. 18.

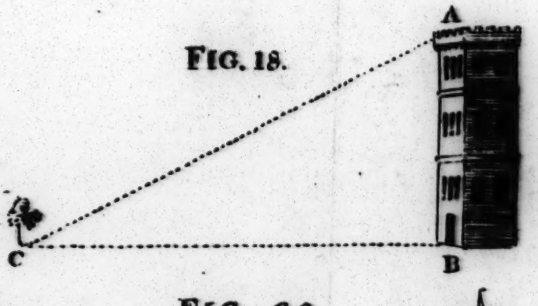



FIG. 19.

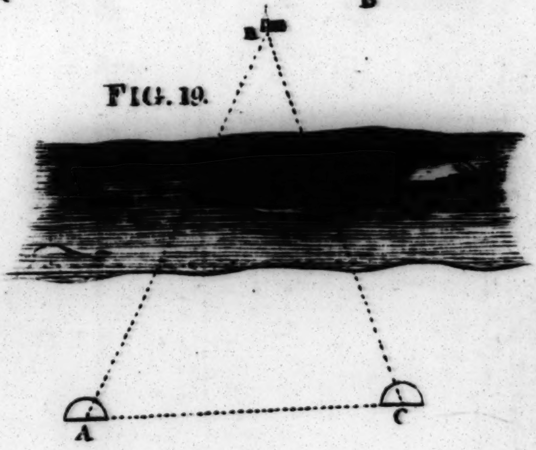



FIG 20

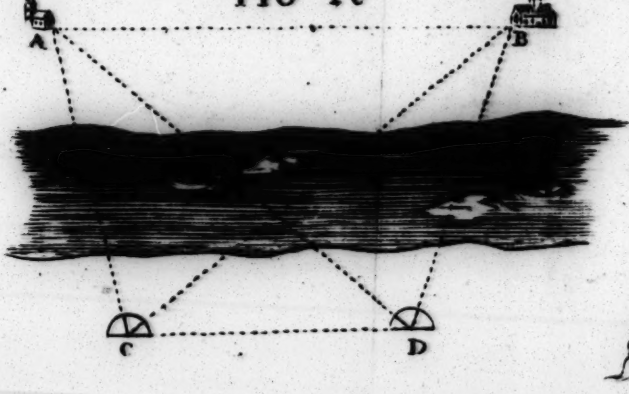
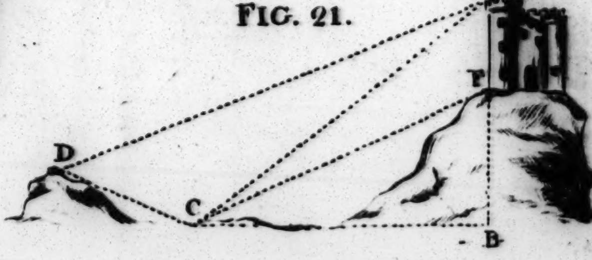



FIG. 21.



the foresaid right line RA and AZ which points to the Zenith.

Let it now be required to measure the angle ACB (Fig. 15.) in any plane, comprehended between the right lines AC and BC, drawn from two points A and B, to the place of station C. Let the Graphometer be placed at C, supported by its *fulcrum* (as was shown above); and let the immoveable sights on the side of the instrument DE be directed towards the point A; and likewise (while the instrument remains immoveable) let the sights of the ruler FG (which is moveable about the centre C) be directed to the point B. It is evident that the moveable ruler cuts off an arch DH, which is the measure of the angle ACB sought. Moreover, by the same method, the inclination of CE, or of FG, may be observed with the meridian line, which is pointed out by the magnetic needle inclosed in the box, and is moveable about the centre

centre of the instrument, and the measure of this inclination or angle found in degrees.

## PROP. XIII. FIG. 16.

*To measure an accessible height by the Geometrical Quadrant.*

BY the the 12th Prop. of this part, let the angle C be found by means of the quadrant. Then in the triangle ABC, right-angled at B, (BC being supposed the horizontal distance of the observator from the tower), having the angle at C, and the side BC, the required height BA will be found by the 3d case of plain trigonometry.

## PROP. XIV. FIG. 17.

*To measure an inaccessible height by the Geometrical Quadrant.*

LET the angle ACB be observed with the quadrant (by the 12th Prop. of this part;) then let the obser-

ver go from C to the ſecond ſtation D, in the right line BCD (providing BCD be a horizontal plane); and after meaſuring this diſtance CD, take the angle ADC likewiſe with the quadrant. Then, in the triangle ACD, there is given the angle ADC, with the angle ACD; becauſe ACB was given before: Therefore (by 32. 1. Eucl.) the remaining angle CAD is given likewiſe. But the ſide CD is likewiſe given, being the diſtance of the ſtation C and D; therefore (by the firſt caſe of oblique-angled triangles in Trigonometry) the ſide AC will be found. Wherefore, in the right-anlged triangle ABC, all the angles and the hypothenuſe AC are given; conſequently, by the 4th caſe of Trigonometry, the height ſought AB will be found; as alſo (if you pleaſe) the diſtance of the ſtation C from AB, the perpendicular within the hill or inacceſſible height.

## PROP. XV. FIG. 18.

*From the top of a given height, to measure the distance* BC.

LET the angle BAC be observed by the 12th of this part; wherefore in the triangle ABC, right-angled at B, there is given by observation the angle at A; whence (by the 32. 1. Eucl.) there will also be given the angle BCA: Moreover the side AB (being the height of the tower) is supposed to be given. Wherefore, by the 3d case of Trigonometry, BC, the distance sought, will be found.

## PROP. XVI. FIG. 19.

*To measure the distance of two places* A *and* B, *of which one is accessible, by the Graphometer.*

LET there be erected at two points A and C, sufficiently distant, two visible signs; then (by the 12th of this) let

let the two angles BAC, BCA be taken by the Graphometer. Let the diſtance of the ſtations A and C be meaſured with a chain. Then the third angle B being known, and the ſide AC being likewiſe known; therefore, by the firſt caſe of Trigonometry, the diſtance required, AB, will be found.

### PROP. XVII. Fig. 20.

*To meaſure by the Graphometer, the diſtance of two places, neither of which is acceſſible.*

LET two ſtations C and D be choſen, from each of which the places may be ſeen whoſe diſtance is ſought: Let the angles ACD, ACB, BCD, and likewiſe the angles BDC, BDA, CDA, be meaſured by the Graphometer; let the diſtance of the ſtations C and D be meaſured by a chain, or (if it be neceſſary) by the preceeding practice. Now, in the triangle ACD, there are given two angles ACD and ADC; therefore the

the third CAD is likewiſe given; Moreover the ſide CD is given; therefore, by the firſt caſe of Trigonometry, the ſide AD will be found. After the ſame manner, in the triangle BCD, from all the angles and one ſide CD given, the ſide BD is found. Wherefore, in the triangle ADB, from the given ſides DA and DB, and the angle ADB contained by them, the ſide AB (the diſtance ſought) is found by the 4th caſe of Trigonometry of oblique-angled triangles.

Let it be noted, that it is not neceſſary that the points A, B, C, and D, be in one plane; and that any triangle is in one plane, by 2d Prop. 11th of Eucl.

PROP.

## PROP. XVIII. FIG. 21.

*It is required by the Graphometer and Quadrant, to measure an accessible height* AB, *placed so on a steep, that one can neither go near it in an horizontal plane, nor recede from it, as we supposed in the solution of the* 14*th* Prop.

LET there be chosen any situation as C, and another D; where let some mark be erected: Let the angles ACD and ADC be found by the Graphometer; then the third angle DAC will be known. Let the side CD, the distance of the stations, be measured with a chain, and thence (by Trigon.) the side AC will be found. Again, in the triangle ACB, right-angled at B, having found by the Quadrant the angle ACB, the other angle CAB is known likewise: But the side AC in the triangle ADC is already known; therefore the height required AB will be found

found by the 4th caſe of right-angled triangles. If the height of the tower is wanted, the angle BCF will be found by the Quadrant; which being taken from the angle ACB already known, the angle ACF will remain: But the angle FAC was known before; therefore the remaining angle AFC will be known. But the ſide AC was alſo known before; therefore, in the triangle AFC, all the angles and one of the ſides AC being known, AF, the height of the tower above the hill, will be found by Trigonometry.

## *SCHOLIUM.*

It were eaſy to add many other methods of meaſuring heights and diſtances; but, if what is above be underſtood, it will be eaſy (eſpecially for one that is verſed in the elements) to contrive methods for this purpoſe, according to the occaſion: So that there is no need of adding any more of

this

this ſort. We ſhall ſubjoin here a method by which the diameter of the earth may be found out.

## PROP. XIX. Fig. 22.

*To find the diameter of the earth from one obſervation.*

LET there be choſen a high hill A B, near the ſea-ſhore, and let the obſervator on the top of it, with an exact Quadrant divided into minutes and ſeconds by tranſverſe diviſions, and fitted with a teleſcope in place of the common ſights, meaſure the angle ABE contained under the right line AB, which goes to the centre, and the right line BE drawn to the ſea, a tangent to the globe at E; let there be drawn from A perpendicular to B D, the line AF meeting BE in F. Now in the right-angled triangle BAF all the angles are given, alſo the ſide AB, the height of the hill; which is

to be found by some of the foregoing methods, as exactly as possible; and by Trigonometry, the sides BF and AF are found. But, by Corol. 36. 3. Eucl. AF is equal to FE; therefore BE will be known. Moreover, by 36th, 3. Eucl. the rectangle under BA and BD, is equal to the square of BE. And thence, by 17th, 6. Eucl. as AB : BE : : BE : BD. Therefore, since AB and BE are already given, BD will be found by 11th, 6. Eucl. or by the rule of three; and subtracting BA, there will remain AD the diameter of the earth sought.

### *SCHOLIUM.*

Many other methods might be proposed for measuring the diameter of the earth. The most exact in my opinion is that proposed by Mr *Picart*, of the academy of sciences at *Paris*. But since it does not belong to this place, we refer you to the philosophical transactions, where you will find it described.

" According

"According to Mr *Picart*, a degree
" of the meridian at the latitude of 49°
" 21', was 57,060 *French Toises*, each of
" which contains six feet of the same
" measure; from which it follows, that,
" if the earth be an exact sphere, the
" circumference of a great circle of it
" will be 123,249,600 *Paris* feet, and the
" semidiameter of the earth, 19,615,800
" feet: But the *French* Mathematicians,
" who of late have examined Mr *Pi-*
" *cart's* operations, assure us, That the
" degree in that latitude is 57,183 *Toi-*
" *ses*. They measured a degree in *Lap-*
" *land*, in the latitude of 66° 20', and
" found it of 57,438 *Toises*. By com-
" paring these degrees, as well as by the
" observations on pendulums, and the
" theory of gravity, it appears that the
" earth is an oblate spheroid; and (sup-
" posing those degrees to be accurately
" measured) the axis or diameter that
" passes through the poles will be to
" the diameter of the equator, as 177

"to 178, or the earth will be 22 miles "higher at the equator than at the "poles. A degree has likewise been "measured at the equator, and found "to be considerably less than at the la-"titude of *Paris*; which confirms the "oblate figure of the earth. But an "account of this last mensuration has "not been published as yet. If the "earth was of an uniform density from "the surface to the centre, then, ac-"cording to the theory of gravity, the "meridian would be an exact ellipsis, "and the axis would be to the diame-"ter of the equator as 230 to 231; "and the difference of the semidiame-"ter of the equator and semiaxis about "17 miles."

In what follows, a figure is often to be laid down on paper, like to another figure given; and because this likeness consists in the equality of their angles, and in the sides having the same proportion to each other (by the defini-

tions

tions of the 6th of Eucl.) we are now to ſhew what methods practical Geometricians uſe for making on paper an angle equal to a given angle, and how they conſtitute the ſides in the ſame proportion. For this purpoſe they make uſe of a Protractor, (or, when it is wanting, a Line of chords), and of a Line of equal parts.

## PROP. XX.

FIG. 23. 24. 25. 26. and 27.

*To deſcribe the conſtruction and uſe of the Protractor, of the Line of chords, and of the Line of equal parts.*

THE Protractor is a ſmall ſemicircle of braſs, or ſuch ſolid matter. The ſemicircumference is divided into 180 degrees. The uſe of it is, to draw angles on any plane, as on paper, or to examine the extent of angles already laid down. For this laſt purpoſe, let

let the ſmall point in the centre of the Protractor be placed above the angular point, and let the ſide AB coincide with one of the ſides that contain the angle propoſed; the number of degrees cut off by the other ſide, computing on the Protractor from B, will ſhow the quantity of the angle that is to be meaſured.

But if an angle is to be made of a given quantity on a given line, and at a given point of that line, let AB coincide with the given line, and let the centre A of the inſtrument be applied to that point. Then let there be a mark made at the given number of degrees; and a right line drawn from that mark to the given point, will conſtitute an angle with the given right line, of the quantity required; as is manifeſt.

This is the moſt natural and eaſy method, either for examining the extent of an angle on paper, or for deſcribing

on

on paper an angle of a given quantity.

But when there is ſcarcity of inſtruments, or becauſe a line of chords is more eaſily carried about, (being deſcribed on a ruler on which there are many other lines beſides), practical Geometricians frequently make uſe of it. It is made thus; Let the quadrant of a circle be divided into 90 degrees; (as in Fig. 24.). The line AB is the chord of 90 degrees; the chord of every arch of the quadrant is transferred to this line AB, which is always marked with the number of degrees in the correſponding arch.

Note, That the chord of 60 degrees is equal to the radius, by Corol. 15. 4th Eucl. If now a given angle EDF is to be meaſured by the Line of chords from the centre D, with the diſtance DG, (the chord of 60 degrees), deſcribe the arch GF; and let the points G and F be marked where this arch interſects

the

the ſides of the angle. Then if the diſtance GF, applied on the line of chords from A to B, gives (for example) 25 degrees, this ſhall be the meaſure of the angle propoſed.

When an obtuſe angle is to be meaſured with this line, let its complement to a ſemicircle be meaſured, and thence it will be known. It were eaſy to transfer to the diameter of a circle the chords of all arches to the extent of a ſemicircle; but ſuch are rarely found marked upon rules.

But now, if an angle of a given quantity, ſuppoſe of 50 degrees, is to be made at a given point M of the right line KL (FIG. 26.) From the centre M, and the diſtance MN, equal to the chord of 60 degrees, deſcribe the arch QN. Take off an arch NR, whoſe chord is equal to that of 50 degrees on the Line of chords; join the points M and R; and it is plain that MR ſhall contain

contain an angle of 50 degrees with the line KL propofed.

But fometimes we cannot produce the fides, till they be of the length of a chord of 60 degrees on our fcale; in which cafe it is fit to work by a circle of proportions (that is a Sector), by which an arch may be made of a given number of degrees to any radius.

The quantities of angles are likewife determined by other lines ufually marked upon rules, as the lines of fines, tangents, and fecants; but, as thefe methods are not fo eafy or fo proper in this place, we omit them.

To delineate figures fimilar or like to others given, befides the equality of the angles, the fame proportion is to be preferved among the fides of the figure that is to be delineated, as is among the fides of the figures given. For which purpofe, on the rules ufed by artifts, there is a line divided into equal parts, more or lefs in number, and greater or leffer in quantity,

fity, according to the pleaſure of the maker.

A foot is divided into inches; and an inch, by means of tranſverſe lines, into 100 equal parts; ſo that with this ſcale, any number of inches, below twelve, with any part of an inch, can be taken by the compaſſes, providing ſuch part be greater than the one hundredth part of an inch. And this exactneſs is very neceſſary in delineating the plans of houſes, and in other caſes.

## PROP. XXI. Fig. 28.

*To lay down on paper, by the Protractor or line of chords, and line of equal parts, a right-lined figure like to one given, providing the angles and ſides of the figure given be known by obſervation or menſuration.*

FOR example, ſuppoſe that it is known that in a quadrangular figure, one ſide is of 235 feet, that the angle

angle contained by it and the ſecond ſide is of 84°, the ſecond ſide of 288 feet, the angle contained by it and the third ſide of 72°, and that the third ſide is 294 feet. Theſe things being given, a figure is to be drawn on paper like to this quadrangular figure. On your paper, at a proper point A, let a right line be drawn, upon which take 235 equal parts, as AB. The part repreſenting a foot is taken greater or leſſer, according as you would have your figure greater or leſs. In the adjoining figure, the 100th part of an inch is taken for a foot. And accordingly an inch divided into 100 parts, and annexed to the figure, is called a ſcale of 100 feet. Let there be made at the point B (by the preceeding Prop.) an angle ABC of 85°, and let BC be taken of 288 parts like to the former. Then let the angle BCD be made of 72°, and the ſide CD of 294 equal parts. Then let the ſide AD be drawn;

and it will compleat the figure like to the figure given. The meafures of the angle A and D can be known by the protractor or line of chords, and the fide AD by the line of equal parts; which will exactly anfwer to the correfponding angles and to the fide of the primary figure.

After the very fame manner, from the fides and angles given, which bound any right-lined figure, a figure like to it may be drawn, and the reft of its fides and angles be known.

## *COROLLARY.*

Hence any trigonometrical problem in right-lined triangles, may be refolved by delineating the triangle from what is given concerning it, as in this propofition. The unknown fides are examined by a line of equal parts, and the angles by a protractor or line of chords.

PROP.

## PROP. XXII. PROB.

*The diameter of a circle being given, to find its circumference nearly.*

THE periphery of any polygon inſcribed in the circle is leſs than the circumference, and the periphery of any polygon deſcribed about a circle is greater than the circumference. Whence Archimedes firſt diſcovered that the diameter was in proportion to the circumference, as 7 to 22 nearly; which ſerves for common uſe. But the moderns have computed the proportion of the diameter to the circumference to greater exactneſs. Suppoſing the diameter 100, the periphery will be more than 314, but leſs than 315 *. But Ludolphus van Cuelen exceeded the labours of all; for by immenſe ſtudy he found, that, ſuppoſing the

* The diameter is more nearly to the circumference, as 113 to 355.

the diameter

100,000,000.000,000,000.000,000,000.000,000,

the periphery will be leſs than

314,159,265,358,979,323.846,264.338.327,951,

but greater than

314,159,265,358,979,323.846,264,338,327,950;

whence it will be eaſy, any part of the circumference being given in degrees and minutes, to aſſign it in parts of the diameter.

*Of ſurveying and meaſuring of* LAND.

HITHERTO we have treated of the meaſuring of angles and ſides, whence it is abundantly eaſy to lay down a field, a plane, or an entire country: For to this nothing is requiſite but the protraction of triangles, and of other plain figures, after having meaſured their ſides and angles. But as this is eſteemed an important part of practical Geometry, we ſhall ſubjoin here an account of it with all poſſible brevity; ſuggeſting withal, that a ſur-

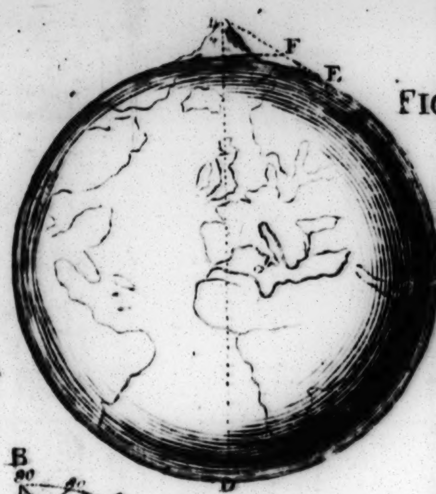
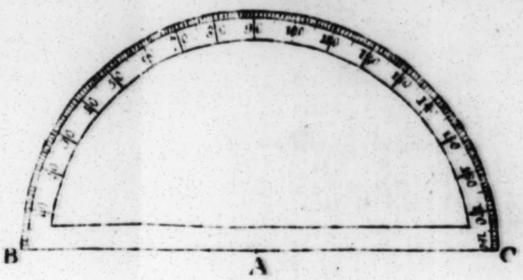


FIG. 22.

FIG. 23.



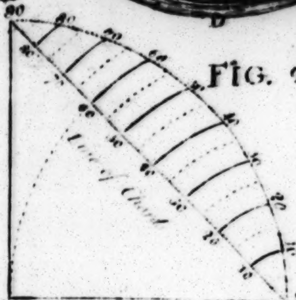


FIG. 24.

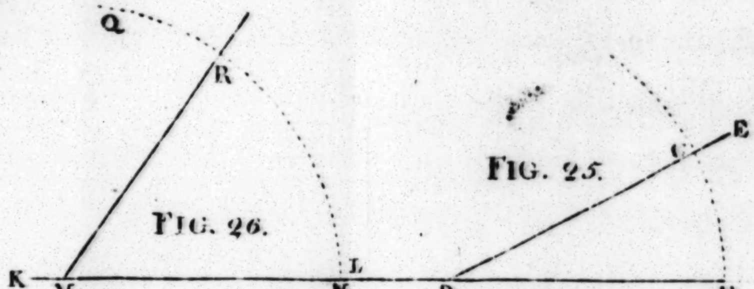


FIG. 26.

FIG. 25.

Scale of half an Inch divided into 100 parts

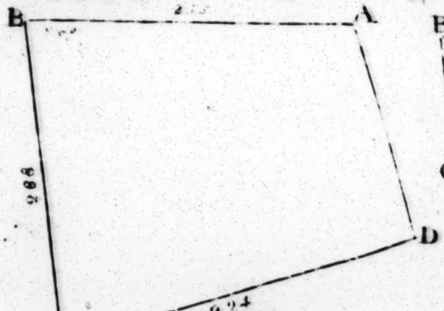


FIG. 28

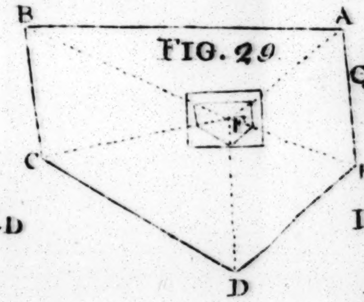


FIG. 29

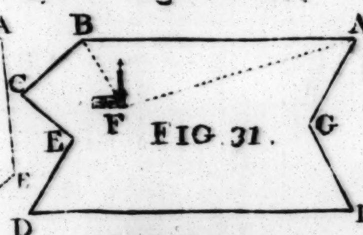


FIG. 31.

FIG. 32.

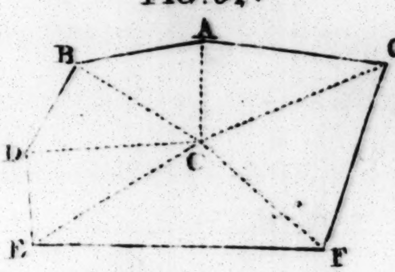
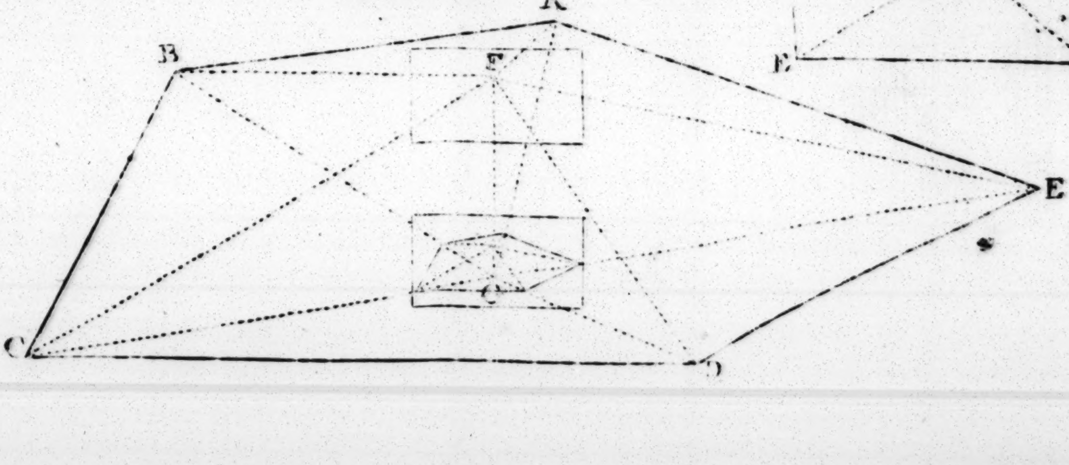


FIG. 30.



veyor will improve himſelf more by one day's practice, than by a great deal of reading.

## PROP. XXIII. Prob.

*To explain what Surveying is, and what inſtruments Surveyors uſe.*

FIRST, it is neceſſary that the ſurveyor view the field that is to be meaſured, and inveſtigate its ſides and angles, by means of an iron chain (having a particular mark at each foot of length, or at any number of feet, as may be moſt convenient for reducing lines or ſurfaces to the received meaſures *), and the Graphometer deſcribed above. *Secondly*, It is neceſſary to delineate the field *in plano*, or to form a map of it; that is, to lay down on paper a figure ſimilar to the field; which is done by the Protractor

* See above p. 4 the account of Gunter's chain, and of the chain that is moſt convenient for meaſuring land in Scotland.

Protractor ( or line of chords) and of the line of equal parts. *Thirdly*, It is necessary to find out the area of the field so surveyed and represented by a map. Of this last we are to treat below, in the second part.

The sides and angles of small fields are surveyed by the help of a plain table; which is generally of an oblong rectangular figure, and supported by a *fulcrum*, so as to turn every way by means of a ball and socket. It has a moveable frame, which surrounds the board, and serves to keep a clean paper put on the board close and tight to it. The sides of the frame facing the paper are divided into equal parts every way. The board hath besides a box with a magnetic needle, and moreover a large index with two sights, On the edge of the frame of the board are marked degrees and minutes, so as to supply the room of a Graphometer.

PROP.

## PROP XXIV.

### PROB. FIG. 29.

*To delineate a field by the help of a plain-table, from one ſtation whence all its angles may be ſeen, and their diſtances meaſured by a chain.*

LET the field that is to be laid down be ABCDE. At at any convenient place F, let the plain-table be erected; cover it with clean paper, in which let ſome point near the middle repreſent the ſtation. Then applying at this place the index with the ſights, direct it ſo as that through the ſights ſome mark may be ſeen at one of the angles, ſuppoſe A; and from the point F, repreſenting the ſtation, draw a faint right line along the ſide of the index: then, by the help of the chain, let FA the diſtance of the ſtation from the

the foresaid angle be measured. Then taking what part you think convenient for a foot or pace from the line of equal parts, set off on the faint line the parts corresponding to the line FA that was measured; and let there be a mark made representing the angle of the field A. Keeping the table immoveable, the same is to be done with the rest of the angles; then right lines joining those marks shall include a figure like to the field, as is evident from 5. 6. Eucl.

## *COROLLARY.*

The same thing is done in like manner by the Graphometer; for having observed in each of the triangles, AFB, BFC, CFD, &c. the angle at the station F, and having measured the lines from the station to the angles of the field, let similar triangles be protracted on paper (by the 21. of this) having their common vertex in the point of station.

All

All the lines, excepting thoſe which repreſent the ſides of the field, are to be drawn faint or obſcure.

Note 1. When a Surveyor wants to ay down a field, let him place diſtinctly in a regiſter all the obſervations of the angles, and the meaſures of the ſides, until, at time and place convenient, he draw out the figure on paper.

Note 2. The obſervations made by the help of the Graphometer are to be examined; for all the angles about the point F ought to be equal to four right ones, by 13th, 1 Eucl.

## PROP. XXV.

### PROB. FIG. 30.

*To lay down a field by means of two stations, from each of which all the angles can be seen, by measuring only the distance of the stations.*

LET the instrument be placed at the station F; and having chosen a point representing it upon the paper which is laid upon the plain table, let the index be applied at this point, so as to be moveable about it. Then let it be directed successively to the several angles of the field; and when any angle is seen through the sights, draw an obscure line along the side of the index. Let the index, with the sights, be directed after the same manner to the station G; on the obscure line drawn along its side, pointing to A, set off from the scale of equal parts a line corresponding to the measured

meaſured diſtance of the ſtations; and this will determine the point G. Then remove the inſtrument to the ſtation G; and applying the index to the line repreſenting the diſtance of the ſtations, place the inſtrument ſo that the firſt ſtation may be ſeen through the ſights. Then the inſtrument remaining immoveable, let the index be applied at the point repreſenting the ſecond ſtation G; and be ſucceſſively directed by means of its ſights, to all the angles of the field, drawing (as before) obſcure lines; and the interſection of the two obſcure lines that were drawn to the ſame angle from the two ſtations will always repreſent that angle on the plan. Care muſt be taken that thoſe lines be not miſtaken for one another. Lines joining thoſe interſections will form a figure on the paper like to the field.

SCHO-

## SCHOLIUM.

It will not be difficult to do the ſame by the graphometer, if you keep a diſtinct account of your obſervations of the angles made by the line joining the ſtations, and the lines drawn from the ſtations to the reſpective angles of the field. And this is the moſt common manner of laying down whole countries. The tops of two mountains are taken for two ſtations, and their diſtance is either meaſured by ſome of the methods mentioned above, or is taken according to common repute. The ſights are ſucceſſively directed towards cities, churches, villages, forts, lakes, turnings of rivers, woods, &c.

Note, The diſtance of the ſtations ought to be great enough, with reſpect to the field that is to be meaſured; ſuch ought to be choſen as are not in a line with any angle of the field. And care ought to be taken likewiſe that the angles, for example, FAG, FDG, &c. be neither

neither vey acute, nor very obtuſe. Such angles are to be avoided as much as poſſible; and this admonition is found very uſeful in practice.

## PROP. XXVI.

### Prob. Fig. 31.

*To lay down any field, however irregular its figure may be, by the help of the Graphometer.*

LET ABCEDHG be ſuch a field. Let its angles (in going round it) be obſerved with a graphometer (by the 12th of this) and noted down; let its ſides be meaſured with a chain; and (by what was ſaid on the 21ſt of this) let a figure like to the given field be protracted on paper. If any mountain is in the circumference, the horizontal line hid under it is to be taken for a ſide, which may be found by two or three obſervations according to ſome of the

the methods deſcribed above; and its place on the map is to be diſtinguiſhed by a ſhade, that it may be known a mountain is there.

If not only the circumference of the field is to be laid down in the plan, but alſo its contents, as villages, gardens, churches, public roads, we muſt proceed in this manner.

Let there be (for example) a church F, to be laid down in the plan. Let the angles ABF, BAF be obſerved and protracted on paper in their proper places, the interſection of the two ſides BF and AF will give the place of the church on the paper: Or, more exactly, the lines BF, AF being meaſured, let circles be deſcribed from the centres B and A, with parts from the ſcale correſponding to the diſtances BF and AF, and the place of the church will be at their interſection.

Note 1. While the angles obſerved by the graphometer are taken down, you muſt

muſt be careful to diſtinguiſh the external angles, as E and G, that they may be rightly protracted afterwards on paper.

Note 2. Our obſervations of the angles may be examined by computing if all the internal angles make twice as many right angles, four excepted, as there are ſides of the figure: for this is demonſtrated by 32d, 1. Eucl. But in place of any external angle DEC, its complement to a circle is to be taken.

## PROP. XXVII.

### PROB. FIG. 32.

*To lay down a plain field without inſtruments.*

IF a ſmall field is to be meaſured, and a map of it to be made, and you are not provided with inſtruments; let it be ſuppoſed to be divided into triangles, by right-lines, as in the figure; and after meaſuring

meaſuring the three ſides of any of the triangles, for example of ABC, let its ſides be laid down from a convenient ſcale on paper, by the 22d of this. Again, let the other two ſides BD, CD of the triangle CBD be meaſured and protracted on the paper by the ſame ſcale as before. In the ſame manner proceed with the reſt of the triangles of which the field is compoſed, and the map of the field will be perfected; for the three ſides of a triangle determine the triangle; whence each triangle on the paper is ſimilar to its correſpondent triangle in the field, and is ſimilarly ſituated: conſequently the whole figure is like to the whole field.

## *SCHOLIUM.*

If the field be ſmall, and all its angles may be ſeen from one ſtation, it may be very well laid down by the plain-table by the 24th of this. If the field be

be larger, and have the requiſite conditions, and great exactneſs is not expected, it likewiſe may be plotted by means of the plain-table, or by the Graphometer, according to the 25th of this; but in fields that are irregular and mountainous, when an exact map is required, we are to make uſe of the Graphometer, as in the 26th of this, but rarely of the plain-table.

Having protracted the bounding lines, the particular parts contained within them may be laid down by the proper operations for this purpoſe, delivered in the 26th propoſition; and the method deſcribed in the 27th propoſition may be ſometimes of ſervice; for we may truſt more to the meaſuring of ſides, than to the obſerving of angles. We are not to compute four-ſided and many-ſided figures till they are reſolved into triangles: for the ſides do not determine thoſe figures.

In the laying down of cities, or the like, we may make uſe of any of the methods deſcribed above that may be moſt convenient.

The map being finiſhed, it is tranſferred on clean paper, by putting the firſt ſketch above it, and marking the angles by the point of a ſmall needle. Theſe points being joined by right lines, and the whole illuminated by colours proper to each part, and the figure of the mariner's compaſs being added to diſtinguiſh the north and ſouth, with a ſcale on the margin, the map or plan will be finiſhed and neat.

We have thus brieflyand plainly treated of ſurveying, and ſhown by what inſtruments it is performed; having avoided thoſe methods which depend on the magnetic needle, not only becauſe its direction may vary in different places of a field (the contrary of this at leaſt doth not appear, but becauſe the quantity of an angle obſerved by it cannot be exactly

ly known; for an error of two or three degrees can ſcarcely be avoided in taking angles by it. As for the remaining part of ſurveying, whereby the area of a field already laid down on paper is found in acres, roods, or any other ſuperficial meaſures; this we leave to the following part, which treats of the menſuration of ſurfaces.

" Beſides the inſtruments deſcribed
" above, a ſurveyor ought to be pro-
" vided with an off-ſet ſtaff equal in
" length to ten links of the chain, and
" divided into ten equal parts. He
" ought likewiſe to have ten arrows or
" ſmall ſtraight ſticks near two feet
" long, ſhod with iron ferrils. When
" the chain is firſt opened, it ought to
" be examined by the off-ſet ſtaff. In
" meaſuring any line, the leader of the
" chain is to have the ten arrows at
" firſt ſetting out. When the chain is
" ſtretched in the line, and the near
" end

" end touches the place from which
" you meaſure, the leader ſticks one of
" the ten arrows in the ground, at the
" far end of the chain. Then the lead-
" er leaving the arrow, proceeds with
" the chain another length; and the
" chain being ſtretched in the line, ſo
" that the near end touches the firſt
" arrow, the leader ſticks down another
" arrow at his end of the chain. The
" line is preſerved ſtraight, if the ar-
" rows be always ſet ſo as to be in a
" right line with the place you mea-
" ſure from, and that to which you are
" going. In this manner they proceed
" till the leader have no more arrows.
" At the eleventh chain, the arrows
" are to be carried to him again, and
" he is to ſtick one of them into the
" ground, at the end of the chain. And
" the ſame is to be done at the 21. 31.
" 41. *&c.* chains, if there are ſo many
" in the right line to be meaſured. In
" this manner you can hardly commit
" an

"an error in numbering the chains, un-
"less of ten chains at once.

"The Off-set staff serves for measuring readily the distances of any things proper to be represented in your plan, from the station-line while you go along. These distances ought to be entered into your field-book, with the corresponding distances from the last station, and proper remarks, that you may be enabled to plot them justly, and be in no danger of mistaking one for another, when you extend your plan. The field-book may be conveniently divided into five columns. In the middle-column the angles at the several stations taken by the Theodolite are to be entered, with the distances from the stations. The distances taken by the Off-set staff, on either side of the station-line, are to be entered into columns on either side of the middle-column, according to their position

"with

" with respect to that line. The names " or characters of the objects, with pro- " per remarks, may be entered in co- " lumns on either side of these last.

" Because, in the place of the Gra- " phometer described by our author, " Surveyors now make use of the The- " odolite, we shall subjoin a description " of Mr Sisson's latest improved Theodo- " lite from Mr Gardner's practical Sur- " veying improved. See a figure of it in " plate 4.

" In this instrument, the three staffs, " by brass ferrils at top, screw into bell- " metal joints, that are moveable be- " tween brass pillars, fixed in a strong " brass plate; in which, round the cen- " tre, is fixed a socket with a ball move- " able in it, and upon which the four " screws press, that set the limb hori- " zontal: Next above is another such " plate, through which the said screws " pass, and on which, round the centre, " is fixed a frustum of a cone of bell-

" metal,

" metal, whoſe axis (being connected
" with the centre of the ball) is always
" perpendicular to the limb, by means
" of a conical braſs ferril fitted to it,
" whereon is fixed the compaſs-box; and
" on it the limb, which is a ſtrong bell-
" metal ring, whereon are moveable three
" braſs indexes; in whoſe plate are fixed
" four braſs pillars, that, joining at top,
" hold the centre pin of the bell-metal
" double ſextant, whoſe double index is
" fixed on the centre of the ſame plate:
" Within the double ſextant is fixed the
" ſpirit level, and over it the teleſcope.

" The compaſs-box is graved with two
" diamonds for North and ſouth, and
" with 20 degrees on both ſides of each,
" that the needle may be ſet to the vari-
" ation, and its error alſo known.

" The limb has two *Fleurs de luce*
" againſt the diamonds in the box, in-
" ſtead of 180 each; and is curiouſly
" divided into whole degrees, and num-
" bered to the left hand at every ten to

twice

" twice 180, having three indexes diſtant
" 120, (with Nonius's diviſions on each
" for the decimals of a degree), that are
" moved by a pinion fixed below one of
" them, without moving the limb; and
" in another is a ſcrew and ſpring under,
" to fix it to any part of the limb. It
" has alſo diviſions numbered, for taking
" the quarter girt in inches of round tim-
" ber at the middle height, when ſtand-
" ing ten feet horizontally diſtant from
" its centre; which at 20 muſt be dou-
" bled, and at 30 tripled; to which a
" ſhorter index is uſed, having Nonius's
" diviſions for the decimals of an inch;
" but an abatement muſt be made for the
" bark, if not taken off.

" The double ſextant is divided on
" one ſide from under its centre (when
" the ſpirit-tube and teleſcope are le-
" vel) to above 60 degrees each way,
" and numbered at 10, 20, &c. and
" the

" the double index, (through which it is " moveable) ſhews on the ſame ſide the " degree and decimal of any altitude or " depreſſion to that extent by Nonius's " diviſions: On the other ſide are divi- " ſions numbered, for taking the up- " right height of timber, &c. in feet, " when diſtant 10 feet; which at 20 muſt " be doubled, and at 30 tripled; and al- " ſo the quantities for reducing hypo- " thenuſal lines to horizontal. It is " moveable by a pinion fixed in the dou- " ble index.

" The teleſcope is a little ſhorter than " the diameter of the limb, that a fall " may not hurt it; yet it will magnify " as much, and ſhew a diſtant object as " perfect, as moſt of triple its length. In " its focus are very fine croſs wires, whoſe " interſection is in the plane of the dou- " ble ſextant; and this was a whole cir- " cle, and turned in a lathe to a true " plane, and is fixed at right angles to " the limb; ſo that, whenever the limb

" is ſet horizontal, (which is readily done
" by making the ſpirit-tube level over
" two ſcrews, and the like over the other
" two), the double ſextant and teleſcope
" are moveable in a vertical plane; and
" then every angle taken on the limb
" (though the teleſcope be never ſo much
" elevated or depreſſed) will be an angle
" in the plane of the horizon. And this
" is abſolutely neceſſary in plotting a ho-
" rizontal plane.

" If the lands to be plotted are hilly,
" and not in any one plane, the lines
" meaſured cannot be truly laid down
" on paper, without being reduced to
" one plane, which muſt be the horizon-
" tal, becauſe angles are taken in that
" plane.—

" In viewing my objects, if they have
" much altitude or depreſſion, I either
" write down the degree and decimal
" ſhewn on the double ſextant, or the
" links ſhewn on the back-ſide; which

" laſt

"last subtracted from every chain in the "station-line, leaves the length in the "horizontal plane. But if the degree is "taken, the following table will shew the "quantity.

*A TABLE of the links to be subtracted out of every chain in* hypothenusal lines *of several degrees altitude, or depression, for reducing them to* horizontal.

| Degrees. | Links. | Degrees. | Links. | Degrees. | Links. |
|---|---|---|---|---|---|
| 4,05 | $\frac{1}{4}$ | 14,07 | 3 | 23,074 | 8 |
| 5,73 | $\frac{1}{2}$ | 16,26 | 4 | 24,495 | 9 |
| 7,02 | $\frac{3}{4}$ | 18,195 | 5 | 25,84 | 10 |
| 8,11 | 1 | 19,95 | 6 | 27,13 | 11 |
| 11,48 | 2 | 21,565 | 7 | 28,36 | 12 |

"Let the first station-line really mea- "sure 1107 links, and the angle of alti- "tude or depression be 19°, 95; looking "in the table I find against 19°,95, is 6 "links. Now 6 times 11 is 66; which "subtracted from 1107, leaves 1041, the

"true

" true length to be laid down in the
" plan.

" It is uſeful in ſurveying, to take the
" angles, which the bounding lines form,
" with the magnetic needle, in order to
" check the angles of the figure, and to
" plot them conveniently afterwards."

## PART II.

### *Of the ſurfaces of bodies.*

THE ſmalleſt ſuperficial meaſure with us is a ſquare inch; 144 of which make a ſquare foot. Wrights make uſe of theſe in the meaſuring of deals and planks; but the ſquare foot which the glaziers uſe in meaſuring of glaſs, conſiſts only of 64 ſquare inches. The other meaſures are, *firſt*, the ell ſquare; 2*dly*, the fall, containing 36 ſquare ells; 3*dly*, the rood, containing 40 falls; 4*thly*, the acre, containing 4 roods,

roods. Slaters, masons, and pavers, use the ell square and the fall; surveyors of land use the square ell, the fall, the rood, and the acre.

The superficial measures of the English are, *first*, the square foot; *2dly*, the square yard, containg 9 square feet; for their yard contains only 3 feet; *3dly*, the pole, containing $30\frac{1}{4}$ square yards; *4thly*, the rood, containing 40 poles; *5thly*, the acre, containing 4 roods. And hence it is easy to reduce our superficial measures to the English, or theirs to ours.

"In order to find the content of a "field, it is most convenient to mea-"sure the lines by the chains described "above, p. 4. that of 22 yards for com-"puting the English acres, and that of "24 Scots ells for the acres of Scotland. "The chain is divided into 100 links, "and the square of the chain is 10,000 "square links; ten squares of the "chain, or 100,000 square links, give "an

" an acre. Therefore if the area be ex-
" pressed by square links, divide by
" 100,000, or cut off five decimal pla-
" ces, and the quotient shall give the
" area in acres and decimals of an a-
" cre. Write the entire acres apart; but
" multiply the decimals of an acre by
" 4, and the product shall give the re-
" mainder of the area in roods and deci-
" mals of a rood. Let the entire roods
" be noted apart after the acres; then
" multiply the decimals of a rood by 40,
" and the product shall give the remain-
" der of the area in falls or poles. Let
" the entire falls or poles be then writ
" after the roods, and multiply the de-
" cimals of a fall by 36, if the area is
" required in the measures of Scotland;
" but multiply the decimals of a pole by
" $30\frac{1}{4}$, if the area is required in the mea-
" sures of England, and the product
" shall give the remainder of the area
" in square ells in the former case, but
" in square yards in the latter. If, in
" the

" the former caſe, you would reduce the
" decimals of the ſquare ell to ſquare
" feet, multiply them by 9.50694; but
" in the latter caſe, the decimals of the
" Engliſh ſquare yard are reduced to
" ſquare feet, by multiplying them by 9.

" Suppoſe, for example, that the area
" appears to contain 12.65842 ſquare
" links of the chain of 24 ells; and that
" this area is to be expreſſed in acres,
" roods, falls, &c. of the meaſures of
" Scotland. Divide the ſquare-links by
" 100,000, and the quotient 12.65842
" ſhows the area to contain 12 acres
" $\frac{65842}{100000}$ of an acre. Multiply the
" decimal part by 4, and the product
" 2.63368 gives the remainder in roods
" and decimals of a rood. Thoſe de-
" cimals of the rood being multiplied
" by 40, the product gives 25.3472 falls.
" Multiply the decimals of the fall by
" 36, and the product gives 12.4992
" ſquare ells. The decimals of the
" ſquare ell multiplied by 9.50994
" give

"give 4.7458 ſquare feet. Therefore "the area propoſed amounts to 12 "acres, 2 roods, 25 falls, 12 ſquare ells, "and 4 $\frac{7458}{10000}$ ſquare feet.

"But if the area contains the ſame num-"ber of ſquare links of Gunter's chain, "and is to be expreſſed by Engliſh mea-"ſures, the acres and roods are compu-"ted in the ſame manner as in the former "caſe. The poles are computed as the "falls. But the decimals of the pole, "viz. $\frac{3472}{10000}$, are to be multiplied by 30$\frac{1}{4}$ (or 30.25), and the product gives 10.5028 ſquare yards. The decimals of "the ſquare yard multipled by 9, give "4.5252 ſquare feet; therefore in this "caſe the area is in Engliſh meaſure 12 "acres, 2 roods, 25 poles, 10 ſquare "yards, and 4 $\frac{5252}{10000}$ ſquare feet.

"The Scots acre is to the Engliſh acre, "by ſtatute, as 100,000 to 78,694, if "we have regard to the difference be-"twixt the Scots and Engliſh foot above

"mentioned.

"mentioned. But it is cuſtomary in "ſome parts of England to have 18,21, "&c. feet to a pole, and 160 ſuch poles "to an acre; whereas, by the ſtatute, "$16\frac{1}{2}$ feet make a pole. In ſuch caſes "the acre is greater in the duplicate ra- "tio of the number of feet to a pole.

"They who meaſure land in Scotland "by an ell of 37 Engliſh inches, make "the acre leſs than the true Scots acre "by $593\frac{6}{10}$ ſquare Engliſh feet, or by "about $\frac{1}{93}$ of the acre.

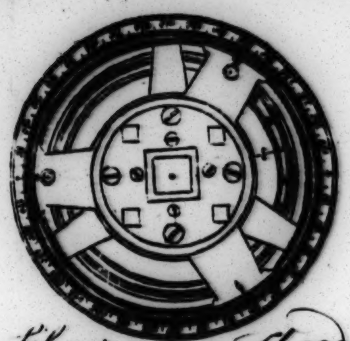
"An huſband-land contains 6 acres "of ſock and ſythe-land, that is of land "that may be tilled with a plough, and "mown with a ſythe; 13 acres of arable "land make an oxgang or oxengate; "four oxengate make a pound-land of "old extent (by a decree of the Exche- "quer, March 11. 1585), and is called "called *librata terræ*. A forty ſhilling "land of old extent contains eight ox- "gang, or 104 acres.

 "The

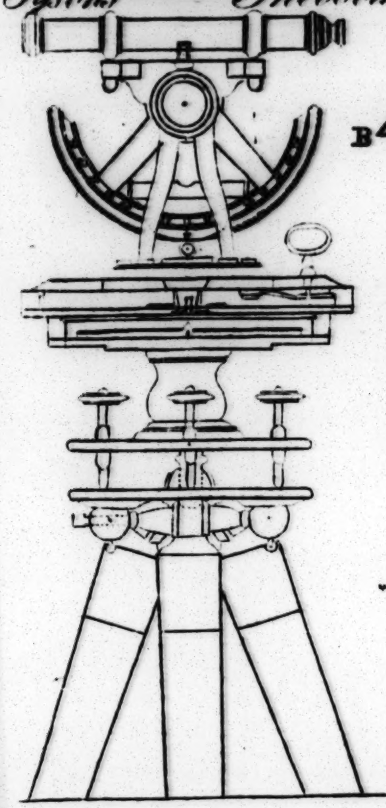
" The Arpent about Paris contains " 32400 ſquare Paris feet, and is equal " to $2\frac{2}{3}$ Scots roods or $3\frac{37}{100}$ Engliſh " roods.

" The *Actus quadratus*, according to " Varro, Collumella, &c. was a ſquare of " 120 Roman feet. The Jugerum was " the double of this. 'Tis to the Scots " acre as 10,000 to 20,456, and to the " Engliſh acre as 10,000 to 16,097. It " was divided (like the As) into 12 unciæ, " and the uncia into 24 ſcrupula." This, with the three preceeding paragraphs, are taken from an ingenious manuſcript written by Sir Robert Stewart profeſſor of natural philoſphy. The greateſt part of the table in p. 6. was taken from it likewiſe.

PROP.



Sisson's Theodolite.



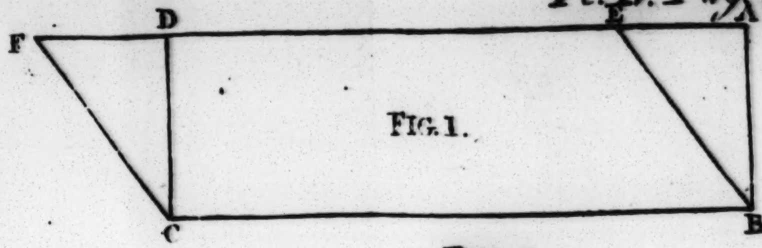



FIG. 1.

FIG. 2.

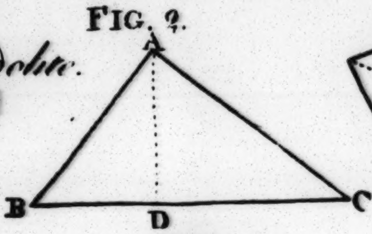



FIG. 3.

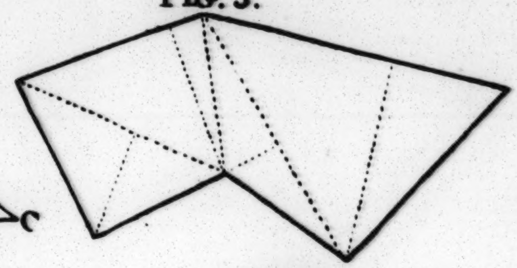


FIG. 4.

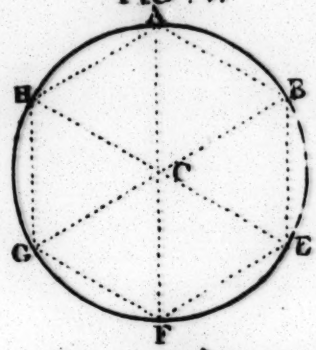
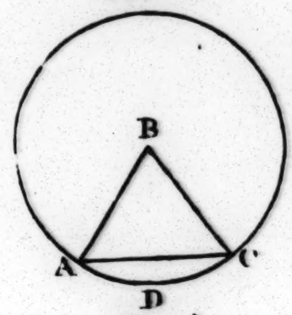



FIG. 5.



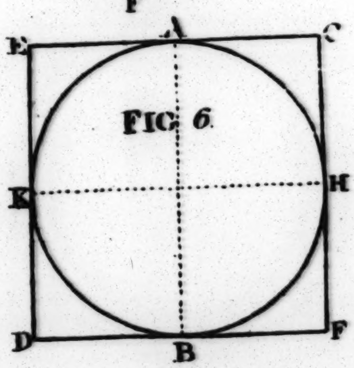




FIG. 6.

FIG. 7.

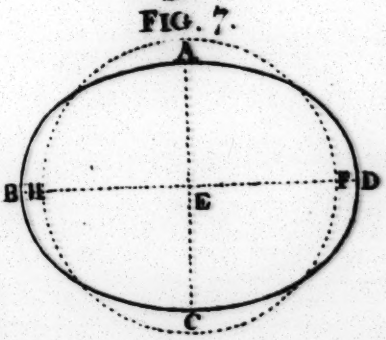



FIG. 8.

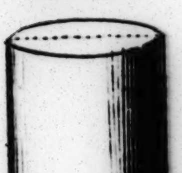


FIG. 9.

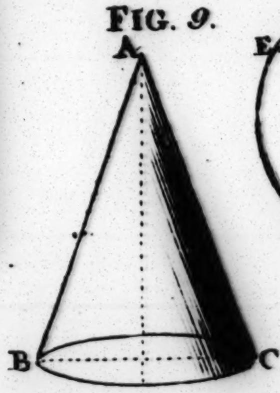



FIG. 10.

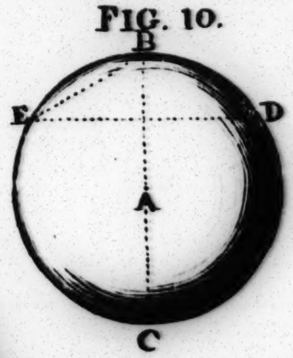
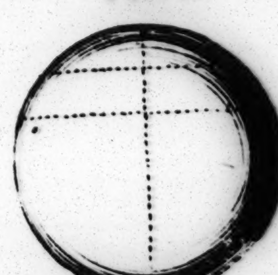



FIG. 11.



## PROP. I.

### PROB. FIG. I.

*To find out the area of a rectangular parallelogram* ABCD.

LET the side AB, for example, be five feet long, and BC (which constitutes with BA a right angle at B) be 17 feet. Let 17 be multiplied by 5, and the product 85 will be the number of square feet in the area of the figure ABCD. But if the parallelogram proposed is not rectangular as BEFC, its base BC multiplied into its perpendicular height AB (not into its side BE) will give its area. This is evident from 35th 1. Eucl.

PROP.

## PROP. II.

### PROB. FIG. 2.

*To find the area of a given triangle.*

LET the triangle BAC be given, whose base BC is supposed 9 feet long; Let the perpendicular AD be drawn from the angle A opposite to the base, and let us suppose AD to be four feet. Let the half of the perpendicular be multiplied into the base, or the half of the base into the perpendicular, or take the half of the product of the whole base into the perpendicular, the product gives 18 square feet for the area of the given triangle.

But if only the sides are given, the perpendicular is found either by protracting the triangle, or by 12th and 13th 2. Eucl. or by trigonometry. But how the area of a triangle may be found from the given sides only, shall be shewn in the 4th prop. of this part.

PROP.

## PROP. III.

### PROB. FIG. 3.

*To find the area of any rectilineal figure.*

IF the figure be irregular, let it be resolved into triangles; and drawing perpendiculars to the bases in each of them, let the area of each triangle be found by the preceeding Prop. and the sum of these areas will give the area of the figure.

### *SCHOLIUM* 1.

In measuring boards, planks, and glass, their sides are to be measured by a foot-rule divided into 100 equal parts; and after multiplying the sides, the decimal fractions are easily reduced to lesser denominations. The mensuration of these is easy, when they are rectangular parallelograms.

*SCHO-*

## *SCHOLIUM* 2.

If a field is to be meaſured, let it firſt be plotted on paper, by ſome of the methods deſcribed in the preceeding part, and let the figure ſo laid down be divided into triangles, as was ſhown in the preceeding propoſition.

The baſe of any triangle, or the perpendicular upon the baſe, or the diſtance of any two points of the field, is meaſured by applying it to the ſcale according to which the map is drawn.

## *SCHOLIUM* 3.

But if the field given be not in a horizontal plane, but uneven and mountainous, the ſcale gives the horizontal line between any two points, but not their diſtance meaſured on the uneven ſurface of the field. And indeed it would appear that the horizontal plane is to be accounted the area of an uneven and hilly country. For if ſuch ground is laid out

for

for building on, or for planting with trees, or bearing corn, ſince theſe ſtand perpendicular to the horizon, it is plain that a mountainous country cannot be conſidered as of greater extent for thoſe uſes than the horizontal plane; nay, perhaps, for nouriſhing of plants, the horizontal plane may be preferable.

If however the area of a figure, as it lies irregularly on the ſurface of the earth, is to be meaſured, this may be eaſily done by reſolving it into triangles as it lies. The ſum of their areas will be the area ſought; which exceeds the area of the horizontal figure more or leſs, according as the field is more or leſs uneven.

PROP.

## PROP. IV.

### PROB. FIG. 2.

*The sides of a triangle being given, to find the area, without finding the perpendicular.*

LET all the sides of the triangle be collected into one sum; from the half of which let the sides be separately subtracted, that three differences may be found betwixt the foresaid half sum and each side; then let these three differences and the half sum be multiplied into one another, and the square root of the product will give the area of the triangle. For example, let the sides be 10, 17, 21; the half of their sum is 24; the three differences betwixt this half sum and the three sides, are 14, 7, and 3. The first being multiplied by the second, and their product by the third, we have 294 for the product of the

the differences; which multiplied by the foresaid half sum 24, gives 7056; the square root of which 84 is the area of the triangle. The demonstration of this, for the sake of brevity, we omit. It is to be found in several treatises, particularly in Clavius's Practical Geometry.

## PROP. V.

### THEOR. FIG. 4.

*The area of the ordinate figure* ABEFGH *is equal to the product of the half circumference of the polygon, multiplied into the perpendicular drawn from the centre of the circumscribed circle to the side of the polygon.*

FOR the ordinate figure can be resolved into as many equal triangles, as there are sides of the figure; and since each triangle is equal to the product of half the base into the perpendicular,

it is evident that the sum of all the triangles together, that is the polygon, is equal to the product of half the sum of the bases (that is the half of the circumference of the polygon) into the common perpendicular height of the triangles drawn from the centre C to one of the sides; for example to AB.

## PROP. VI.

### PROB. FIG. 5.

*The area of a circle is found by multiplying the half of the periphery into the radius, or the half of the radius into the periphery.*

FOR a circle is not different from an ordinate or regular polygon of an infinite number of sides, and the common height of the triangles into which the polygon or circle may be supposed to be divided, is the radius of the circle. Were

Were it worth while, it were easy to demonstrate accurately this proposition, by means of the inscribed and circumscribed figures, as is done in the 5th Prop. of the treatise of Archimedes concerning the dimensions of the circle.

## *COROLLARY.*

Hence also it appears that the area of the sector ABCD is produced, by multiplying the half of the arch into the radius; and likewise that the area of the segment of the circle ADC is found, by subtracting from the area of the sector the area of the triangle ABC.

## PROP. VII.

### THEOR. FIG. 6.

*The circle is to the square of the diameter, as 11 to 14 nearly.*

FOR if the diameter AB be supposed to be 7, the circumference AHBK will

will be almost 22 (by the 22d Prop. of the first part of this), and the area of the square DC will be 49; and, by the preceeding prop. of this, the area of the circle will be $38\frac{1}{2}$: Therefore the square DC will be to the inscribed circle as 49 to $38\frac{1}{2}$, or as 98 to 77, that is, as 14 to 11. *Q. E. D.*

If greater exactness is required, you may proceed to any degree of accuracy: For the square DC is to the inscribed circle, as 1 to $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}$ &c. *in infinitum*.

"This series will be of no service for "computing the area of the circle ac"curately, without some further arti"fice, because it converges at too slow "a rate. The area of the circle will "be found exactly enough for most "purposes, by multiplying the square "of the diameter by 7854, and divi"ding by 10,000, or cutting off four "decimal places from the product; "for the area of the circle is to the "circumscribed

"circumſcribed ſquare nearly as 7854 to "10,000."

## PROP. VIII.

### PROB. FIG. 7.

*To find the area of a given ellipſe.*

LET ABCD be an ellipſe, whoſe greater diameter is BD, and leſſer AC, biſecting the greater perpendicularly in E. Let a mean proportional HF he found (by 13th 6. Eucl.) between AC and BD, and (by the 6th of this) find the area of the circle deſcribed on the diameter HF. I ſay, that this area is equal to the area of the ellipſe ABCD. For becauſe, as BD to AC, ſo the ſquare of BD to the ſquare of HF, (by 2. Cor. 20th 6. Eucl.): But (by the 2d 12. Eucl.) as the ſquare of BD to the ſquare of HF, ſo is the circle of the diameter BD to the circle of the diameter HF : Therefore as BD to AC, ſo is the circle of the diameter

meter BD to the circle of the diameter HF. And (by the 5th Prop. of Archimedes of ſpheroids) as the greater diameter BD to the leſſer AC, ſo is the circle of the diameter BD to the ellipſe ABCD. Conſequently (by the 11th 5. Eucl.) the circle of the diameter BD will have the ſame proportion to the circle of the diameter HF, and to the ellipſe ABCD. Therefore, by 9th 5. Eucl. the area of the circle of the diameter HF will be equal to the area of the ellipſe ABCD. *Q. E. D.*

## *S C H O L I U M.*

From this and the two preceeding propoſitions, a method is derived of finding the area of an ellipſe. There are two ways: 1ſt, Say, as one is to the leſſer diameter, ſo is the greater diameter to a fourth number, (which is found by the rule of three). Then again ſay, as 14 to 11, ſo is the 4th number found to the area

area fought. But the fecond way is fhorter. Multiply the leffer diameter into the greater, and the product by 11; then divide the whole product by 14, and the quotient will be the area fought of the ellipfe. For example, Let the greater diameter be 10, and the leffer 7, by multiplying 10 by 7, the product is 70; and multiplying that by 11, it is 770; and dividing 770 by 14, the quotient will be 55, which is the area of the ellipfe fought.

" The area of the ellipfe will be found " more accurately, by multiplying the " product of the two diameters by 7854."

We fhall add no more about other plain furfaces, whether rectilinear or curvilinear, which feldom occur in practice; but fhall fubjoin fome propofitions about meafuring the furfaces of folids.

PROP.

## PROP. IX. PROB.

*To measure the surface of any prism.*

BY the 14th definition of the 11th Eucl. a prism is contained by planes, of which two opposite sides (commonly called the bases) are plain rectilineal figures; which are either regular and ordinate, and measured by Prop. 5. of this part; or however irregular, and then they are measured by the 3d Prop. of this book. The other sides are parallelograms, which are measured by the 1st Prop. of the second part; and the whole superficies of the prism consists of the sum of those taken altogether.

## PROP. X. PROB.

*To measure the superficies of any pyramid.*

SINCE its basis is a rectilineal figure, and the rest of the plains terminating in the top of the pyramid are triangles;

angles; theſe meaſured ſeparately, and added together, give the ſurface of the pyramid required.

## PROP. XI. PROB.

*To meaſure the ſuperficies of any regular body.*

THESE bodies are called regular, which are bounded by aequilateral and aequiangular figures. The ſuperficies of the tetraedron conſiſts of four equal and aequiangular triangles; the ſuperficies of the hexaedron, or cube, of ſix equal ſquares; an octedron, of eight equal aequilateral triangles; a dodecaedron, of twelve equal and ordinate pentagons; and the ſuperficies of an icoſiædron, of twenty equal and aequilateral triangles. Therefore it will be eaſy to meaſure theſe ſurfaces from what has been already ſhown.

In the ſame manner we may meaſure the ſuperficies of a ſolid contained by any planes.

## PROP. XII.

### PROB. FIG. 8.

*To measure the superficies of a cylinder.*

BECAUSE a cylinder differs very little from a prism, whose opposite planes (or bases) are ordinate figures of an infinite number of sides, it appears that the superficies of a cylinder, without the bases, is equal to an infinite number of parallelograms; the common altitude of all which is, with the height of the cylinder, and the bases of them all differ very little from the periphery of the circle which is the base of the cylinder. Therefore this periphery multiplied into the common height, gives the superficies of the cylinder, excluding the bases; which are to be measured separately by the help of the 6th Prop. of this part.

This proposition concerning the measure of the surface of the cylinder (excluding

cluding its basis) is evident from this, That when it is conceived to be spread out, it becomes a parallelogram, whose base is the periphery of the circle of the base of the cylinder stretched into a right line, and whose height is the same with the height of the cylinder.

## PROP. XIII.

### PROB. FIG. 9.

*To measure the surface of a right cone.*

THE surface of a right cone is very little different from the surface of a right pyramid, having an ordinate polygon for its base of an infinite number of sides; the surface of which (excluding the base) is equal to the sum of the triangles. The sum of the bases of these triangles is equal to the periphery of the circle of the base, and the common height of the triangles is the side of the cone AB: Wherefore the sum of these triangles

gles is equal to the product of the ſum of the baſes (*i. e.* the periphery of the baſe of the cone) multiplied into the half of the common height, or it is equal to the product of the periphery of the baſe.

If the area of the baſe is likewiſe wanted, it is to be found ſeparately by the 6th Prop. of this part. If the ſurface of a cone is ſuppoſed to be ſpread out on a plane, it will become a ſector of a circle, whoſe radius is the ſide of the cone; and the arch terminating the ſector is made from the periphery of the baſe. Whence, by Corol. 6. Prop. of this, its dimenſion may be found.

## *COROLLARY.*

Hence it will be eaſy to meaſure the ſurface of a *fruſtum* of a cone cut by a plane parallel to the baſe. As to what relates to the meaſuring of the ſurface of the ſcalenous cone, becauſe it is not very uſeful in practice, we ſhall not deſcribe the

the method; which would carry us beyond the limits of this treatise.

## PROP. XIV.

### PROB. FIG. 10.

*To measure the surface of a given sphere.*

LET there be a sphere, whose centre is A, and let the area of its convex surface be required. Archimedes demonstrates (37. Prop. 1. book of the sphere and cylinder) that its surface is equal to the area of four great circles of the sphere; that is, let the area of the great circle be multiplied by 4, and the product will give the area of the sphere; or, by the 20th 6. and 2d 12. of Eucl. the area of the sphere given is equal to the area of a circle whose radius is the right line BC, the diameter of the sphere. Therefore having measured (by 6th Prop. of this part) the circle described with the radius BC, this will give the surface of the sphere.

## PROP. XV.

### PROB. FIG. 10.

*To measure the surface of a segment of a sphere.*

LET there be a segment cut off by the plane ED. Archimedes demonstrates (49, and 50. 1. *de sphæra*) that the surface of this segment, excluding the circular base, is equal to the area of a circle whose radius is the right line BE drawn from the vertex B of the segment to the periphery of the circle DE. Therefore, by the 6th Prop. of this part, it is easily measured.

### *COROLLARY* 1.

Hence that part of the surface of a sphere that lieth between two parallel planes is easily measured, by subtracting the surface of the lesser segment from the surface of the greater segment.

*COROL-*

## *COROLLARY 2.*

Hence likewiſe it follows, that the ſurface of a cylinder, deſcribed about a ſphere (excluding the baſis) is equal to the ſurface of the ſphere, and the parts of the one to the parts of the other, intercepted between planes parallel to the baſis of the cylinder.

# PART III.

## *Of ſolid figures and their menſuration.*

AS in the preceeding parts we took an inch for the ſmalleſt meaſure in length, and an inch ſquare for the ſmalleſt ſuperficial meaſure; ſo now, in treating of the menſuration of ſolids, we take a cubical inch for the ſmalleſt ſolid meaſure. Of theſe 109 make a Scots pint; other liquid meaſures depend on this, as is generally known.

IN dry meaſures, the firlot, by ſtatute, contains $19\frac{1}{2}$ pints; and on this depend

depend the other dry measures: Therefore, if the content of any solid be given in cubical inches, it will be easy to reduce the same to the common liquid or dry measures, and conversely to reduce these to solid inches. The liquid and dry measures in use among other nations, are known from their writers.

" As to the English liquid measures, " by act of parliament 1706, any round " vessel, commonly called a cylinder, " having an even bottom, being seven " inches in diameter throughout, and " six inches deep from the top of the " inside to the bottom, (which vessel " will be found by computation to contain 230 $\frac{907}{1000}$ cubical inches); or any vessel containing 231 cubical " inches, and no more, is deemed to " be a lawful wine-gallon. An English " pint therefore contains $28\frac{7}{8}$ cubical " inches; two pints make a quart; four " quarts a gallon; 18 gallons a roundlet; three roundlets and an half, or

" 63

FIG. 1.

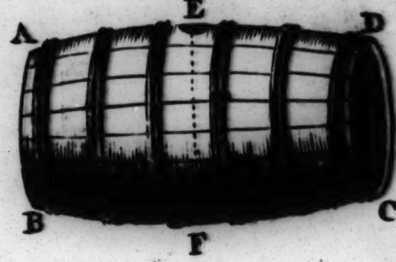



FIG. 2.

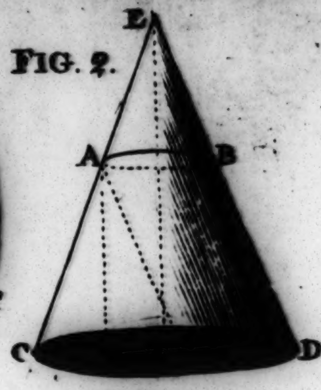



FIG. 3.

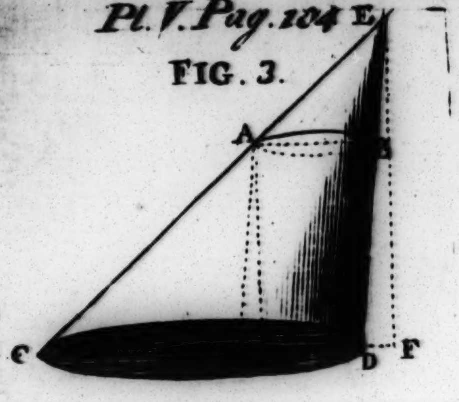



FIG. 4.

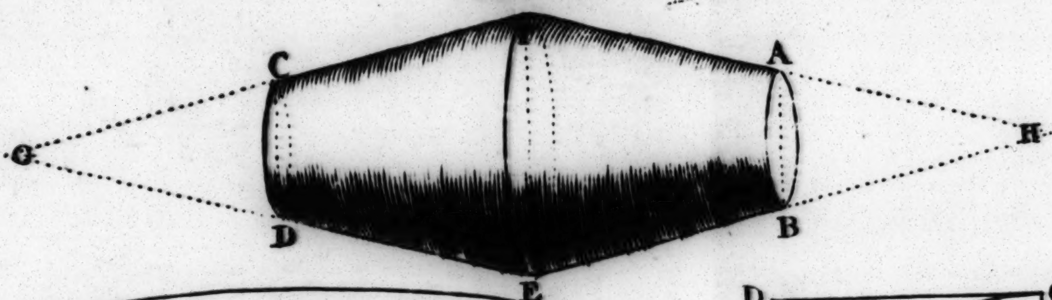



FIG. 5.

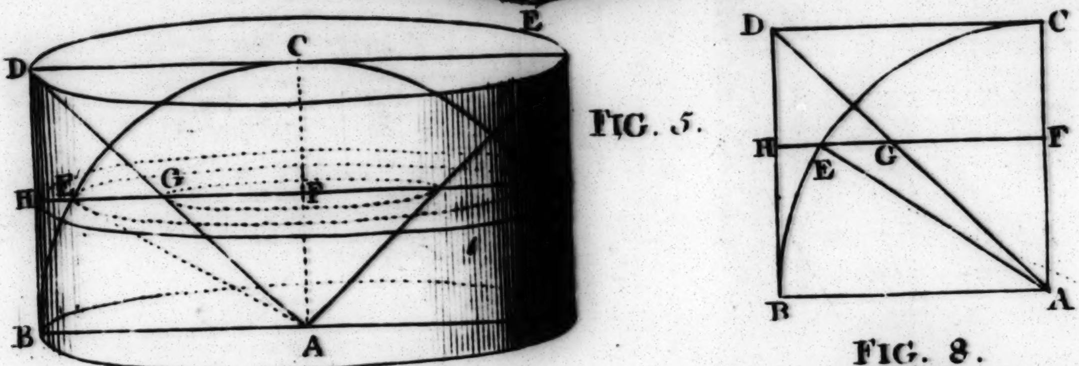



FIG. 8.

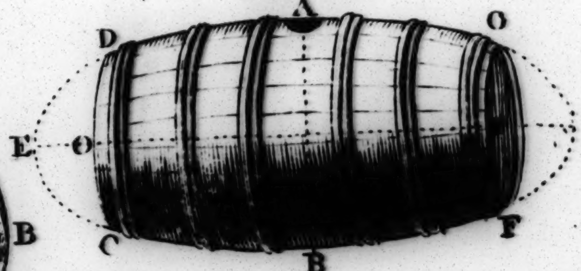



FIG. 7.

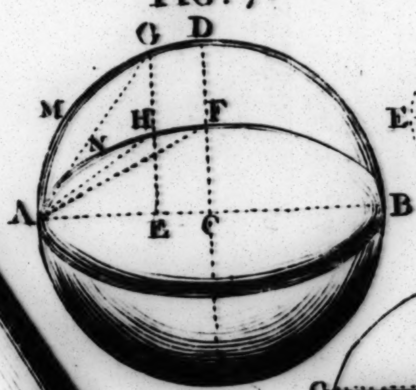



FIG. 6.

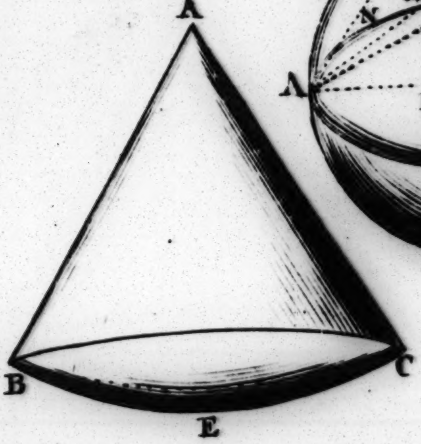



FIG. 9.

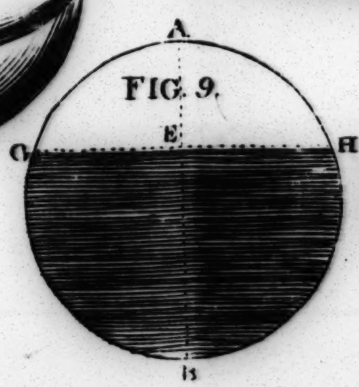
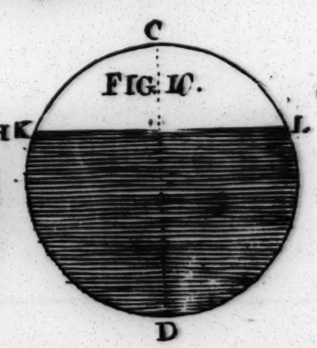



FIG. 10.



"63 gallons, make a hogſhead; the half "of a hogſhead is a barrel; one hogſ-"head and a third, or 84 gallons, "make a puncheon; one puncheon "and a half, or two hogſheads, or 126 "gallons, make a pipe or butt; the "third part of a pipe, or 42 gallons, "make a tierce; two pipes, or three "puncheons, or four hogſheads, make a "ton of wine. Though the Engliſh "wine-gallon is now fixed at 231 cu-"bical inches, the ſtandard kept in "Guildhall being meaſured, before ma-"ny perſons of diſtinction, May 25. "1688, it was found to contain only "224 ſuch inches.

"In the Engliſh beer-meaſure, a gal-"lon contains 282 cubical inches; con-"ſequently $35\frac{1}{4}$ cubical inches make a "pint, two pints make a quart, four "quarts make a gallon, nine gallons a "firkin, four firkins a barrel. In ale, "eight gallons make a firkin, and 32 "gallons make a barrel. By an act "of

" of the first of William and Mary, 34 gal-
" lons is the barrel, both for beer and ale,
" in all places, except within the weekly
" bills of mortality.

" In Scotland, it is known that four
" gills make a mutchkin, two mutch-
" kins make a chopin, a pint is two
" chopins, a quart is two pints, and a
" gallon is four quarts or eight pints.
" The accounts of the cubical inches
" contained in the Scots pint vary con-
" siderably from each other. Accord-
" ing to our author, it contains 109
" cubical inches. But the standard-
" jugs kept by the Dean of Guild of E-
" dinburgh (one of which has the year
" 1555, with the arms of Scotland, and
" of the town of Edinburgh, marked up-
" on it) having been carefully measured
" several times, and by different per-
" sons, the Scots pint, according to those
" standards, was found to contain a-
" bout $103\frac{4}{10}$ cubic inches. The Pew-
" terers jugs (by which the vessels in
"com-

"common uſe are made) are ſaid to "contain ſometimes betwixt 105 and "106 cubic inches. A caſk that was "meaſured by the brewers of Edin-"burgh, before the commiſſioners of "Exciſe in 1707, was found to con-"tain $46\frac{7}{8}$ Scots pints; the ſame veſſel "contained $18\frac{1}{18}$ Engliſh ale-gallons. "Suppoſing this menſurating to be "juſt, the Scots pint will be to the Eng-"liſh ale-gallon as 289 to 750; and if "the Engliſh ale-gallon be ſuppoſed to "contain 282 cubical inches, the Scots "pint will contain 108.664 cubical "inches. But it is ſuſpected, on ſeve-"ral grounds, that this experiment was "not made with ſufficient care and ex-"actneſs.

"The Commiſſioners appointed by "authority of parliament to ſettle the "meaſures and weights, in their act of "February 19. 1618, relate, That having "cauſed fill the Linlithgow firlot with "water, they found that it contained

"$21\frac{1}{4}$

" $21\frac{1}{4}$ pints of the juſt Stirling jug and
" meaſure. They likewiſe ordain that
" this ſhall be the juſt and only firlot,
" and add, *That the wideneſs and bread-*
" *neſs of the which firlot, under and above*
" *even over within the buirds, ſhall contain*
" *nineteen inches and the ſixth part of an inch,*
" *and the deepneſs ſeven inches and a third*
" *part of an inch.* According to this act
" (ſuppoſing their experiment and com-
" putation to have been accurate) the
" pint contained only 99.56 cubical
" inches; for the content of ſuch a
" veſſel as is deſcribed in the act, is
" 2115.85, and this divided by $21\frac{1}{4}$,
" gives 99.56. But, by the weight of
" water ſaid to fill this firlot in the ſame
" act, the meaſure of the pint agrees
" nearly with the Edinburgh ſtandard a-
" bove mentioned.

" As for the Engliſh meaſures of corn,
" the Winchester gallon contains $272\frac{1}{4}$,
" cubical inches, two gallons make a
" peck, four pecks, or eight gallons
" (that

"(that is 2178 cubical inches) make a
"bushel, and a quarter is eight bushels.

"Our author says, that 19½ Scots pints
"make a firlot. But this does not ap-
"pear to be agreeable to the statute a-
"bove mentioned, nor to the standard-
"jugs. It may be conjectured that the
"proportion assigned by him has been
"deduced from some experiment of how
"many pints, according to common use,
"were contained in the firlot. For if we
"suppose those pints to have been each
"of 108.664 cubical inches, according
"to the experiment made in the 1707
"before the commissioners of Excise, de-
"scribed above; then 19½ such pints
"will amount to 2118.94 cubical inches,
"which agrees nearly with 2115.85, the
"measure of the firlot by statute above
"mentioned. But it is probable, that in
"this he followed the act 1587, where it is
"ordained, That the wheat-firlot shall
"contain

"contain 19 pints and two joucattes. A
"wheat-firlot marked with the Linlith-"gow ſtamps being meaſured, was found "to contain about 2211 cubical inches. "By the ſtatute of 1618 the barley-firlot "was to contain 31 pints of the juſt Stir-"ling jug.

"A Paris pint is 48 cubical Paris in-"ches, and is nearly equal to an Engliſh "wine quart. The *Boiſſean* contains "644.68099 Paris cubical inches, or "780.36 Engliſh cubical inches.

"The Roman *Amphora* was a cubical "Roman foot, the *Congius* was the 8th "part of the *Amphora*, the *Sextarius* was "one ſixth of the *Congius*. They di-"vided the *Sextarius* like the *As* or *Li-*"*bra*. Of dry meaſures, the *Medimnus* "was equal to two *Amphoras*, that is, a-"bout $1\frac{1}{2}\frac{1}{5}$ Engliſh legal buſhels; and "the *Modius* was the third part of the "*Amphora*."

PROP.

## PROP. I. PROB.

*To find the solid content of a given prism.*

BY the 2d Prop. of the 2d part of this, let the area of the base of the prism be measured, and be multiplied by the height of the prism, the product will give the solid content of the prism.

## PROP. II. PROB.

*To find the solid content of a given pyramid.*

THE area of the base being found, (by the 3d Prop. of the 2d part), let it be multiplied by the third part of the height of the pyramid, or the third part of the base by the height, the product will give the solid content, by 7th 12. Eucl.

### COROLLARY.

If the solid content of a *frustum* of a pyramid is required, first let the solid

content

content of the entire pyramid be found; from which ſubtract the ſolid content of the part that is wanting, and the ſolid content of the broken pyramid will remain.

## PROP. III. PROB.

*To find the content of a given cylinder.*

THE area of the baſe being found, (by Prop. 6. of the 2d part), if it be a circle, and by Prop. 8. if it be an ellipſe, (for in both caſes it is a cylinder), multiply it by the height of the cylinder, and the ſolid content of the cylinder will be produced.

## *COROLLARY.* FIG. 1.

And in this manner may be meaſured the ſolid content of veſſels and caſks not much different from a cylinder as ABCD. If towards the middle EF it be ſomewhat groſſer, the area of the circle of the baſe being found (by 6th Prop. of the 2d part)

part) and added to the area of the middle circle EF, and the half of their ſum (that is an arithmetical mean between the area of the baſe, and the area of the middle circle) taken for the baſe of the veſſel, and multiplied into its height, the ſolid content of the given veſſel will be produced.

Note, That the length of the veſſel, as well as the diameters of the baſe, and of the circle EF, ought to be taken within the ſtaves; for it is the ſolid content within the ſtaves that is ſought.

## PROP. IV. PROB.

*To find the ſolid content of a given cone.*

LET the area of the baſe (found by Prop. 6. 2d part) be multiplied into $\frac{1}{3}$ of the height, the product will give the ſolid content of the cone; for by 10th 12. Eucl. a cone is the third part of a cylinder that has the ſame baſe and height.

## PROP. V.

### PROB. FIG. 2. and 3.

*To find the solid content of a* frustum *of a cone cut by a plane parallel to the plane of the base.*

FIRST, let the height of the entire cone be found, and thence (by the preceeding Prop.) its solid content; from which subtract the solid content of the cone cut off at the top, there will remain the solid content of the *frustum* of the cone.

How the content of the entire cone may be found, appears thus: Let ABCD be the *frustum* of the cone (either right or scalenous, as in the figures 2. and 3.): Let the cone ECD be supposed to be compleated; Let AG be drawn parallel to DE, and let AH and EF be perpendicular

lar on CD; it will be (by 2d 6. Eucl.) as CG : CA :: CD : CE; but (by the 4th Prop. of the same book) as CA : AH :: CE : EF; consequently (by 22d 5. Eucl.) as CG : AH :: CD : EF; that is, as the excess of the diameter of the lesser base is to the height of the *frustum*, so is the diameter of the greater base to the height of the entire cone.

## *COROLLARY.* FIG. 4.

Some casks whose staves are remarkably bended about the middle, and straight towards the ends, may be taken for two portions of cones, without any considerable error. Thus ABEF is a *frustum* of a right cone, to whose base EF, on the other side, there is another similar *frustum* of a cone joined EDCF. The vertices of these cones, if they be supposed to be compleated, will be found at G and H. Whence, by the preceeding Prop. the solid content of such vessels may be found.

PROP.

## PROP. VI.

### Theor. Fig. 5.

A Cylinder circumſcribed about a ſphere, that is, having its baſe equal to a great circle of the ſphere, and its height equal to the diameter of the ſphere, is to the ſphere as 3 to 2.

Let ABEC be the quadrant of a circle, and ABDC the circumſcribed ſquare; and likewiſe the triangle ADC; by the revolution of the figure about the right line AC, as axis, a hemiſphere will be generated by the quadrant, a cylinder of the ſame baſe and height by the ſquare, and a cone by the triangle. Let theſe three be cut any how by the plane HF, parallel to the baſe AB, the ſection in the cylinder will be a circle whoſe radius is FH, in the hemiſphere a circle of the radius EF, and in the cone a circle of the radius GF.

By

By the 47th 1. Eucl. $EAq$, or $HFq$ $=EFq$ and $FAq$ taken together, (but $AFq$ $=FGq$, becauſe $AC=CD$); therefore the circle of the radius HF is equal to a circle of the radius EF together with a circle of the radius GF; and ſince this is true every where, all the circles together deſcribed by the reſpective *radii* HF (that is, the cylinder) are equal to all the circles deſcribed by the reſpective *radii* EF and FG (that is, to the hemiſphere and the cone taken together); but, by 10th 12. Eucl. the cone generated by the triangle DAC is one third part of the cylinder generated by the ſquare BC. Whence it follows, that the hemiſphere generated by the rotation of the quadrant ABEC is equal to the remaining two third parts of the cylinder, and that the whole ſphere is $\frac{2}{3}$ of the double cylinder circumſcribed about it.

This is that celebrated 39th Prop 1. book of Archimedes of the ſphere and cylinder;

cylinder; in which he determines the proportion of the cylinder to the sphere inscribed to be that of 3 to 2.

## *COROLLARY.*

Hence it follows, that the sphere is equal to a cone whose height is equal to the semidiameter of the sphere, having for its base a circle equal to the superficies of the sphere, or to four great circles of the sphere, or to a circle whose radius is equal to the diameter of the sphere, by 14th Prop. 2d part of this. And indeed a sphere differs very little from the sum of an infinite number of cones that have their bases in the surface of the sphere, and their common vertex in the centre of the sphere; so that the superficies of the sphere, (of whose dimension see 14th Prop. 2d part of this) multiplied into the third part of the semidiameter, gives the solid content of the sphere.

PROP.

## PROP. VII.

### PROB. FIG. 6.

*To find the solid content of a sector of the sphere.*

A Spherical Sector ABC (as appears by the Corol. of the preceeding Prop.) is very little different from an infinite number of cones, having their bases in the superficies of the sphere BEC, and their common vertex in the centre. Wherefore the spherical superficies BEC being found (by 15. Prop. 2d part), and multiplied into the third part of AB the radius of the sphere, the product will give the solid content of the sector ABC.

### *COROLLARY.*

It is evident how to find the solidity of a spherical segment less than a hemisphere, by subtracting the cone ABC from

from the ſector already found. But if the ſpherical ſegment be greater than a hemiſphere, the cone correſponding muſt be added to the ſector, to make the ſegment.

## P R O P. VIII.

### PROB. FIG. 7.

*To find the ſolidity of the ſpheroid, and of its ſegments cut by planes perpendicular to the axis.*

IN the 2d Prop. of this part it is ſhown, that every where EH : EG :: CF : CD; but circles are as the ſquares deſcribed upon their rays, that is, the circle of the radius EH is to the circle of the radius EG, as CF*q* to CD*q*. And ſince it is ſo every where, all the circles deſcribed with the reſpective rays EH, (that is, the ſpheroid made by the rotation of the ſemi-ellipſis AFB around the axis AB) will be to all the circles deſcribed

ſcribed by the reſpective *radii* EG, (that is, the ſphere deſcribed by the rotation of the ſemicircle ADB on the axis AB) as FC*q* to CD*q*; that is, as the ſpheroid to the ſphere on the ſame axis, ſo is the ſquare of the other axis of the generating ellipſe to the ſquare of the axis of the ſphere.

And this holds, whether the ſpheroid be found by a revolution around the greater or leſſer axis.

## *COROLLARY* 1.

Hence it appears, that the half of the ſpheroid, formed by the rotation of the ſpace AHFC around the axis AC, is double of the cone generated by the triangle AFC about the ſame axis; which is the 32d Prop. of Archimedes, of conoids and ſpheroids.

## *COROLLARY* 2.

Hence likewiſe is evident the meaſure of ſegments of the ſpheroid cut by planes

perpendicular to the axis. For the ſegment of the ſpheroid made by the rotation of the ſpace ANHE, round the axis AE, is to the ſegment of the ſphere having the ſame axis AC, and made by the rotation of the ſegment of the circle AMGE, as CF*q* to CD*q*.

But if the meaſure of this ſolid be wanted with leſs labour, by the 34th Prop. of Archimedes, of conoids and ſpheroids, it will be as BE to AC+EB, ſo is the cone generated by the rotation of the triangle AHE round the axis AE, to the ſegment of the ſphere made by the rotation of the ſpace ANHE round the ſame axis AE; which could eaſily be demonſtrated (was this a proper place for it) by the method of indiviſibles.

## *COROLLARY* 3.

Hence it is eaſy to find the ſolid content of the ſegment of a ſphere or ſpheroid intercepted between two parallel planes, perpendicular to the axis. This agrees

agrees as well to the oblate as to the oblong ſpheriod; as is obvious.

*COROLLARY* 4. FIG. 8.

If a caſk is to be valued as the middle piece of an oblong ſpheroid, cut by the two planes DC and FG, at right angles to the axis: Firſt, Let the ſolid content of the half ſpheroid ABCED be meaſured by the preceeding Prop. from which let the ſolidity of the ſegment DEC be ſubtracted, and there will remain the ſegment ABCD; and this doubled will give the capacity of the caſk required.

The following method is generally made uſe of for finding the ſolid content of ſuch veſſels. The double area of the greateſt circle, that is, of that which is deſcribed by the diameter AB at the middle of the caſk, is added to the area of the circle at the end, that is of the circle DC or FG (for they are uſually equal), and the third part of this ſum is

is taken for a mean base of the cask; which therefore multiplied into the length of the cask OP, gives the content of the vessel required.

Sometimes vessels have other figures, different from those we have mentioned; the easy methods of measuring which may be learned from those who practise this art. What hath already been delivered, is sufficient for our purpose.

## PROP. IX.

### PROB. FIG. 9. and 10.

*To find how much is contained in a vessel that is in part empty, whose axis is parallel to the horrizon.*

LET AGBH be the great circle in the middle of the cask, whose segment GBH is filled with liquor, the segment GAH being empty; the segment GBH is known, if the depth EB be known, and

and EH a mean proportional between the segments of the diameter AB and EB; which are found by a rod or ruler put into the vessel at the orifice. Let the basis of the cask, at a medium, be found, which suppose to be the circle CKDL; and let the segment KCL be similar to the segment GAH (which is either found by the rule of three, because as the circle AGBH is to the circle CKDL, so is the segment GAH to the segment KCL; or is found from the tables of segments made by authors); and the product of this segment multiplied by the length of the cask will give the liquid content remaining in the cask.

## PROP. X. PROB.

*To find the solid content of a regular and ordinate body.*

A Tetraedron being a pyramid, the solid content is found by the 2d Prop. of this part. The Hexaedron, or

or cube, being a kind of prifm, it is meafured by the 1ft Prop. of this part. An Octaedron confifts of two pyramids of the fame fquare bafe and of equal heights; confequently its meafure is found from the fecond Prop. of this part. A Dodecaedron confifts of twelve pyramids having equal aequilateral and aequiangular pentagonal bafes; and fo one of thefe being meafured (by 2d Prop of this) and multiplied by 12, the product will be equal to the folid content of the Dodecaedron. The Icofiaedron confifts of 20 equal pyramids having triangular bafes; the folid content of one of which being found (by the 2d Prop. of this) and multiplied by 20, gives the whole folid. The bafes and heights of thefe pyramids, if you want to proceed more exactly, may be found by Trigonometry.

PROP.

## PROP. XI. PROB.

*To find the ſolid content of a body, however irregular.*

LET the given body be immerſed into a veſſel of water, having the figure of a parallelopipedon orpriſm, and let it be noted how much the water is raiſed upon the immerſion of the body. For it is plain that the ſpace which the water fills, after the immerſion of the body, exceeds the ſpace filled before its immerſion, by a ſpace equal to the ſolid content of the body, however irregular. But when this exceſs is of the figure of a parallelopipedon or priſm, it is eaſily meaſured by the firſt Prop. of this part, to wit, by multiplying the area of the baſe, or mouth of the veſſel, into the difference of the elevations of the water before and after immerſion. Whence is found the ſolid content of the body given. *Q. E. I.*

In the ſame way the ſolid content of a part of a body may be found, by immerſing that part only in water.

There

There is no necessity to insist here on diminishing or enlarging solid bodies in a given proportion. It will be easy to deduce these things from the 11th and 12th books of Euclid.

" The following rules are subjoined " for the ready computation of the con- " tents of vessels, and of any solids in " the measures in use in Great Britain.

" I. To find the content of a cylin- " dric vessel in English wine gallons, the " diameter of the base and altitude of " the vessel being given in inches and de- " cimals of an inch.

" Square the number of inches in " the diameter of the vessel; multiply " this square by the number of inches " in the height: then multiply the pro- " duct by the decimal fraction .0034; " and this last product shall give the " content in wine-gallons and decimals " of such a gallon. To express the " rule arithmetically: Let D represent " the

" the number of inches and decimals of
" an inch in the diameter of the vessel,
" and H the inches and decimals of an
" inch in the height of the vessel; then
" the content in wine-gallons shall be
" DDH $\times \frac{34}{10000}$, or DDH $\times$ .0034. *Ex.*
" Let the diameter D=51.2 inches, the
" height H=62.3 inches, then the con-
" tent shall be 51.2 $\times$ 51.2 $\times$ 62.3 $\times$ .0034
" =555.27,342 wine-gallons. This rule
" follows from Prop. 7. of the second
" part, and Prop. 3. of the third part;
" for, by the former, the area of the base
" of the vessel is in square inches DD $\times$
" .7854; and by the latter, the content
" of the vessel in solid inches is DDH $\times$
" .7854; which divided by 231 (the num-
" ber of cubical inches in a wine-gallon)
" gives DDH $\times$ .0034, the content in
" wine-gallons. But though the charges
" in the Excise are made (by statute) on
" the supposition that the wine-gallon
" contains 231 cubical inches; yet it is
" said, that in sale, 224 cubical inches,

R " the

" the content of the ſtandard meaſured in
" Guildhall (as was mentioned above)
" are allowed to be a wine-gallon.

" II. Suppoſing the Engliſh ale-gallon
" to contain 282 cubical inches, the con-
" tent of a cylindric veſſel is computed
" in ſuch gallons, by multiplying the
" ſquare of the diameter of a veſſel by
" its height as formerly, and their pro-
" duct by the decimal fraction .0,027,851.
" That is, the ſolid content in ale-gal-
" lons is DDH × .0,027,851.

" III. Suppoſing the Scots pint to con-
" tain about 103.4 cubical inches, (which
" is the meaſure given by the ſtandards
" at Edinburgh, according to experi-
" ments mentioned above), the content
" of a cylindriac veſſel is computed in
" Scots pints, by multiplying the ſquare
" of the diameter of the veſſel by its
" height, and the product of theſe by the
" decimal fraction .0076. Or the con-

" tent

" tent of ſuch a veſſel in Scots pints is
" DDH × .0076.

" IV. Suppoſing the Wincheſter buſhel " to contain 2187 cubical inches, the " content of a cylindric veſſel is com- " puted in thoſe buſhels by multiply- " ing the ſquare of the diameter of the " veſſel by the height, and the product " by the decimal fraction .0,003,606. " But the ſtandard buſhel having been " meaſured by Mr Everard and others " in 1696, it was found to contain on- " ly 2145.6 ſolid inches; and therefore " it was enacted in the act for laying a " duty upon malt, *That every round bu-* " *ſhel, with a plain and even bottom, being* " $18\frac{1}{2}$ *inches diameter throughout, and* 8 " *inches deep, ſhould be eſteemed a legal* Win- " cheſter *buſhel*. According to this act " (ratified in the firſt year of Queen " Anne) the legal Wincheſter buſhel " contains only 2150.42 ſolid inches. " And the content of a cylindric veſſel " is computed in ſuch buſhels, by mul-

" tiplying

" tiplying the ſquare of the diameter
" by the height, and their product by
" the decimal fraction .0,003,625. Or
" the content of the veſſel in thoſe bu-
" ſhels is DDH × .0,003,625.

" V. Suppoſing the Scots wheat firlot
" to contain 21½ Scots pints, (as is ap-
" pointed by the ſtatute 1618), and the
" pint to be conform to the Edinburgh
" ſtandards above mentioned, the con-
" tent of a cylindric veſſel in ſuch fir-
" lots is computed by multiplying the
" ſquare of the diameter by the height,
" and their product by the decimal
" fraction .00,358. This firlot, in
" 1426, is appointed to contain 17
" pints; in 1457, it was appointed to
" contain 18 pints; in 1587, it is 19¼
" pints; in 1628, it is 21¼ pints: And
" though this laſt ſtatute appears to
" have been founded on wrong compu-
" tations in ſeveral reſpects; yet this
" part of the act that relates to the
" number of pints in the firlot ſeems
" to

" to be the leaſt exceptionable; and " therefore we ſuppoſe the firlot to con- " tain $21\frac{1}{4}$ pints of the Edinburgh ſtan- " dard, or about 2197 cubical inches; " which a little exceeds the Wincheſter " buſhel, from which it may have been " originally copied.

" VI. Suppoſing the bear-firlot to con- " tain 31 Scots pints, (according to the " ſtatute 1618), and the pint conform to " the Edinburgh ſtandards, the content " of a cylindric veſſel in ſuch firlots is " found by multiplying the ſquare of " the diameter by the height, and this " product by .000,245.

" When the ſection of the veſſel is not " a circle, but an ellipſis, the product of " the greateſt diameter by the leaſt, is to " be ſubſtituted in thoſe rules for the " ſquare of the diameter.

" VII. To compute the content of a " veſſel that may be conſidered as a *fru-* " *ſtum* of a cone in any of thoſe mea- " ſures.

" Let

" Let A repreſent the number of in-
" ches in the diameter of the greater
" baſe, B the number of inches in the
" diameter of the leſſer baſe. Compute
" the ſquare of A, the product of A mul-
" tiplied by B, and the ſquare of B, and
" collect theſe into a ſum. Then find the
" third part of this ſum, and ſubſtitute
" it in the preceeding rules in the place
" of the ſquare of the diameter; and
" proceed in all other reſpects as before.
" Thus, for example, the content in wine-
" gallons is $\overline{AA \times AB \times BB} \times \frac{1}{3} \times H \times$
" .0034.

" Or, to the ſquare of half the ſum
" of the diameters A and B, add one
" third part of the ſquare of half their
" difference, and ſubſtitute this ſum
" in the preceeding rules for the ſquare
" of the diameter of the veſſel; for the
" ſquare of $\frac{1}{2}$ A × $\frac{1}{2}$ B added to $\frac{1}{3}$ of the
" ſquare of $\frac{1}{2}$ A—$\frac{1}{2}$ B, gives $\frac{1}{3}$ AA ×
" $\frac{1}{3}$ AB × $\frac{1}{3}$ BB.

" VIII. When

"VIII. When a veſſel is a *fruſtum* of "a parabolic conoid, meaſure the diameter of the ſection at the middle of "the height of the *fruſtum*; and the "content will be preciſely the ſame as of "a cylinder of this diameter, of the "ſame height with the veſſel.

"IX. When a veſſel is a *fruſtum* of a "ſphere, if you meaſure the diameter of "the ſection at the middle of the height "of the *fruſtum*, then compute the content of a cylinder of this diameter of "the ſame height with the veſſel, and "from this ſubtract $\frac{1}{3}$ of the content of "a cylinder of the ſame height, on a "baſe whoſe diameter is equal to its "height; the remainder will give the "content of the veſſel. That is, if D "repreſent the diameter of the middle ſection, and H the height of "the *fruſtum*, you are to ſubſtitute DD "$-\frac{1}{3}$ HH for the ſquare of the diameter of the cylindric veſſel in the firſt "ſix rules.

"X. When

" X. When the vessel is a *frustum* of
" a spheroid, if the bases are equal, the
" content is readily found by the rule
" in p. 120. In other cases let the axis
" of the solid be to the conjugate axis,
" as *n* to 1; Let D be the diameter of
" the middle section of the *frustum*, H
" the height or length of the *frustum*;
" and substitute in the first six rules
" $DD - \frac{HH}{3nn}$ for the square of the square
" of the diameter of the vessel.

" XI. When the vessel is an hyperbo-
" lic conoid, let the axis of the solid be
" to the conjugate axis, as *n* to 1, D
" the diameter of the section at the mid-
" dle of the *frustum*, H the height or
" length: Compute $DD \times \frac{1}{3nn} \times HH$, and
" substitute this sum for the square of
" the diameter of the cylindric vessel in
" the first six rules.

" XII. In general, it is usual to mea-
" sure any round vessel, by distinguish-
" ing it into several *frustums*, and taking
" the diameter of the section at the
" middle

" middle of each *fruſtum*; thence to com-
" pute the content of each, as if it was
" a cylinder of that mean diameter; and
" to give their ſum as the content of
" the veſſel. From the total content, com-
" puted in this manner, they ſubſtract
" ſucceſſively the numbers which expreſs
" the circular areas that correſpond to
" thoſe mean diameters, each as often
" as there are inches in the altitude of
" the *fruſtum* to which it belongs, be-
" ginning with the uppermoſt; and in
" this manner calculate a table for the
" veſſel, by which it readily appears how
" much liquor is at any time contained
" in it, by taking either the dry or wet
" inches; having regard to the inclina-
" tion or drip of the veſſel, when it has
" any.

" This method of computing the con-
" tent of a *fruſtum* from the diameter of
" the ſection at the middle of its height,
" is exact in that caſe only when it is a
" portion of a parabolic conoid; but in

" ſuch veſſels as are in common uſe, the
" error is not conſiderable. When the
" veſſel is a portion of a cone or hyper-
" bolic conoid, the content by this me-
" thod is found leſs than the truth; but
" when it is a portion of a ſphere or
" ſpheroid, the content computed in this
" manner exceeds the truth. The differ-
" ence or error is always the ſame, in the
" different parts of the ſame or of ſimilar
" veſſels, when the altitude of the *fruſtum*
" is given. And when the altitudes are
" different, the error is in the triplicate
" *ratio* of the altitude. If exactneſs be
" required, the error in meaſuring the
" *fruſtum* of a conical veſſel, in this
" manner, is $\frac{1}{4}$ of the content of a cone
" ſimilar to the veſſel, of an altitude
" equal to the height of the *fruſtum*.
" In a ſphere, it is $\frac{1}{3}$ of a cylinder of
" a diameter and height equal to the
" *fruſtum*. In the ſpheroid and hyperbo-
" lic conoid, it is the ſame as in a cone
" generated by the right angled triangle,
" contained

"contained by the two semiaxes of the "figure, revolving about that side which "is the semiaxis of the *frustum*. These "are demonstrated in a treatise of flu-"xions by Mr Colin MacLaurin, p 22. "and 715. where those theorems are "bounded by planes oblique to the axis "in all the solids that are generated by "any conic section revolving about "either axis.

"In the usual method of computing "a table for a vessel, by subducting from "the whole content the number that "expresses the uppermost area, as often "as there are inches in the uppermost "*frustum*, and afterwards the numbers for "the other areas successively; it is ob-"vious that the contents assigned by the "table, when a few of the uppermost "inches are dry, are stated a little too "high, if the vessel stands on its lesser "base, but too low when it stands on "its greater base; because, when one "inch is dry, for example, it is not the

"area

" area at the middle of the uppermost " *frustum*, but rather the area at the mid- " dle of the uppermost inch, that ought " to be subducted from the total content, " in order to find the content in this " case.

" XIII. To measure round timber, Let " the mean circumference be found in " feet and decimals of a foot; square it; " multiply this square by the decimal " .079.577, and the product by the length. " Ex. Let the mean circumference of a " tree be 10.3 feet, and the length 24 feet. " Then 10.3 × 10.3 × 079,577 × 24=202 " .615, is the number of cubical feet in " the tree. The foundation of this rule " is, that when the circumference of a " circle is 1, the area is .0,795,774,715, " and that the areas of circles are as " the squares of their circumferences.

" But the common way used by ar- " tificers for measuring round timber, " differs much from this rule. They " call one fourth part of the circumfe-

" rence

"rence the *girt*, which is by them "reckoned the side of a square, whose "area is equal to the area of the section "of the tree; therefore they square the "*girt*, and then multiply by the length "of the tree. According to their me- "thod, the tree of the last example would "be computed at 159. 13 cubical feet "only.

"How square timber is measured, will "be easily understood from the preceed- "ing Propositions. Fifty solid feet of "hewn timber, and forty of rough tim- "ber, make a load.

"XIV. To find the burden of a ship, "or the number of tons it will carry, "the following rule is commonly given. "Multiply the length of the keel taken "within board, by the breadth of the "ship within board, taken from the mid- "ship beam from plank to plank, and "the product by the depth of the hold, "taken from the plank below the keelson "to the under part of the upper deck

"plank,

"plank, and divide the product by 94, "the quotient is the content of the ton- "nage required. This rule however can- "not be accurate; nor can one rule be "supposed to serve for the measuring "exactly the burden of ships of all sorts. "Of this the reader will find more in "the Memoirs of the Royal Academy of "sciences at Paris for the year 1721.

"Our author having said nothing of "weights, it may be of use to add brief- "ly, that the English Troy-pound con- "tains 12 ounces, the ounce 20 penny- "weight, and the penny-weight 24 "grains; that the Averdupois pound "contains 16 ounces, the ounce 16 drams, "and that 112 pounds is usually called "the hundred weight. It is commonly "supposed that 14 pounds Averdupois "are equal to 17 pounds Troy. Ac- "cording to Mr Everard's experiments, "one pound Averdupois is equal to 14 "ounces 11 penny-weight and 16 grains

"Troy,

" Troy, that is, to 7000 grains; and
" an Averdupois ounce is $437\frac{1}{2}$ grains.
" The Scots Troy pound (which, by the
" ſtatute 1718, was to be the ſame with
" the French) is commonly ſuppoſed
" equal to $15\frac{3}{4}$ ounces Engliſh Troy, or
" 7560 grains. By a mean of ſtandards
" kept by the Dean of Guild of Edin-
" burgh, it is $7599\frac{3}{5}$, or 7600 grains.
" They who have meaſured the weights
" which were ſent from London, after
" the union of the kingdoms, to be the
" ſtandards by which the weights in
" Scotland ſhould be made, have found
" the Engliſh Averdupois pound (from
" a medium of the ſeveral weights) to
" weigh 7000 grains, the ſame as Mr
" Everard; according to which, the
" Scots, Paris, or Amſterdam pound,
" will be to the pound Averdupois as
" 38 to 35. The Scots Troy-ſtone con-
" tains 16 pounds, the pound two marks
" or 16 ounces, an ounce 16 drops, a
" drop

" drop 36 grains. Twenty Scots ounces " make a Tron pound; but becauſe it is " uſual to allow one to the ſcore, the " Tron pound is commonly 21 ounces. " Sir John Skene however makes the " Tron ſtone to contain only 19½ " pounds."

FINIS.